Princeton University Physics Competition
2019 Onsite Examination Grading Rubric

November 17, 2019

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1 Solutions / Grading Rubric

1.1 Question 1 (10 points max.)

(a) The second cylinder, the one that rolls without slipping, will have a greater linear velocity. Give 5 points or 0 points for the answer.

(b) You lose energy to friction in the first case. Give 5 points or 0 points for the explanation.

1.2 Question 2 (15 points max.)

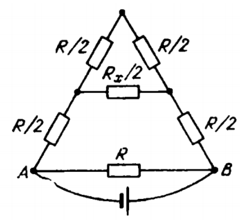

Figure 1:

(a) The figure above gives the equivalent circuit that we want. The current flowing into A from above is the same as the current flowing out of B into the upper branch. Likewise, the current flowing into A from the lowermost horizontal segment is that which left B along the same segment. There is then no reason to have a marked point of wire at the midpoint between A and B, so you can disconnect the arrangement of wire there. By similar arguments, you can disconnect the other vertices except at the midpoints between A and B and the top point of the largest triangle. We expect the total resistance between these points to be exactly half of R_{AB}. Give 7 points, 4 points, or 0 points for the answer. Don't give 7 points unless the work done is entirely correctly. Give 4 points if legitimate progress is made towards the solution, and 0 points if no such progress is made.

(b) For whatever reason Krotov renames $R_{AB} = R_x$. You write down the usual rules for parallel circuits to get

$$\frac{1}{R_x} = \frac{1}{R} + \frac{1}{\left(\frac{1}{R} + \frac{2}{R_x}\right)^{-1} + R} \tag{1}$$

This gives:

$$3{R_x}^2 + 2RR_x - 2R^2 = 0 \quad \longrightarrow \quad R_x = R_{AB} = \frac{\sqrt{7}-1}{3}R. \tag{2}$$

Give 8 points, 4 points, or 0 points for the answer with the same guidelines as in part a). In general, if you are unsure of whether or not to give partial credit or no credit, give partial credit and move on.

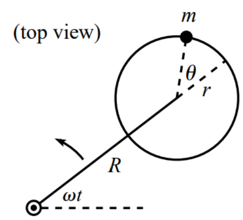

Figure 2:

1.3 Question 3 (15 points max.)

(a) The velocity of the bead is

$$(\dot{x}, \dot{y}) = (-\omega R \sin \omega t - r(\omega + \dot{\theta}) \sin(\omega t + \theta), \omega R \cos \omega t + r(\omega + \dot{\theta}) \cos(\omega t + \theta)). \tag{3}$$

The plane is horizontal, so there is no potential energy term to consider and $\mathcal{L} = T$. We then have

$$\mathcal{L} = \frac{m}{2}((R^2\omega^2 + r^2(\omega + \dot{\theta})^2 + 2Rr\omega(\omega + \dot{\theta}) \cos \theta). \tag{4}$$

Give 5 points, 3 points, or 0 points for the answer. Give 5 points only if everything is correct, and give 3 points if the velocity or position is correct. Give 0 points otherwise.

(b) Use the Euler-Lagrange equation. You get

$$r\ddot{\theta} + Rw^2 \sin \theta = 0. \tag{5}$$

Give 5 points, 3 points, or 0 points. Give 5 points if everything is right, 3 points if they write down the Euler-Lagrange equation, and 0 otherwise.

(c) Equilibrium occurs when $\dot{\theta} = \ddot{\theta} = 0$, so $\theta = 0$ in the equation below.

$$\ddot{\theta} + \left(\frac{R}{r}\right) w^2 \theta = 0 \tag{6}$$

This is the equation of a SHO. The frequency of small oscillations is $\Omega = \omega\sqrt{\frac{R}{r}}$. If $R \ll r$, then $\Omega \approx 0$ since the hoop barely moves. If $R \approx r$, then $\Omega \approx \omega$. If $R \gg r$, then Ω is very large. Give 5 points if everything is correct, 3 points if one case is explained incorrectly or the conclusion is wrong, and 0 points otherwise.

1.4 Question 4 (25 points max.)

(a) Use the parallel axis theorem to get $I = \frac{1}{3}m_r l^2 + m_d l^2 + \frac{1}{2}m_d R^2$. 3 points all or nothing.

(b) The equation of motion should be

$$\ddot{\theta} = \frac{-(\frac{1}{2}m_r + m_d)gl \sin \theta}{(\frac{1}{3}m_r + m_d)l^2 + \frac{1}{2}m_d R^2}. \tag{7}$$

Give 7 points for the correct answer, 4 points for a legitimate attempt at the answer using either torque analysis or the Lagrangian formulism, and 0 points otherwise. If you are unsure if an attempt is legitimate or not, give partial credit of 4 points and move on.

(c) This is immediate from part b). All or nothing 3 points. The answer is of course what precedes the sine term above.

(d) The moment of inertia of the system is smaller, so ω will increase. You can in fact subtract off the $\frac{1}{2}m_d R^2$ term from equation (7). Give 5 points, 3 points, or 0 points. 5 points for the correct answer with the correct justification, 3 points with just the correct answer, and 0 otherwise.

(e) First find the center of mass. Treat the empty space as a negative mass $-\frac{M}{4}$ centered $R/2$ to the right of the origin, which we take as the geometric center of the system. Solving:

$$x_{\text{cm}} = \frac{(M \times 0 - \frac{M}{4} \times \frac{R}{2})}{(M - \frac{M}{4})} = -\frac{R}{6} \tag{8}$$

To find the moment of inertia of the whole system, we can simply apply the parallel axis theorem and sum over the resultant moments of inertia of the single disks. Specifically:

$$I_M^x = I_M^{\text{cm}} + M x_{\text{cm}}^2 \qquad I_{-\frac{M}{4}}^x = I_{-\frac{M}{4}}^{\text{cm}} - \frac{M}{4}(x_{\text{cm}} + \frac{R}{2})^2 \tag{9}$$

$$I_{\text{tot}}^{\text{cm}} = I_M^x + I_{-\frac{M}{4}}^x \tag{10}$$

Given that the moment of inertia from the center of mass of a disk of mass m and radius r is equal to $\frac{1}{2}mr^2$, it follows, after just a bit of algebra, that:

$$I_{\text{tot}}^{\text{cm}} = \frac{15}{32} M R^2 \tag{11}$$

1.5 Question 5 (10 points max.)

As stated in the exam booklet, making an honest attempt at 1 and 2 will earn students 6 points, and making an honest attempt at 1,2, and 3 will earn students 8 points. Entirely valid reasoning that addresses 1,2, and 3 will receive all 10 points.

For specific examples of possible alternatives that students may provide, please see pages 6-8 of Everett's paper found at https://www-tc.pbs.org/wgbh/nova/manyworlds/pdf/dissertation.pdf