Physics Unlimited Premier Competition
2019 Onsite Examination

November 17, 2019

Competitor ID (Exam Code): ______________________________
Initials: ____

This exam contains 8 pages (including this cover page) and 5 questions worth 75 points total.
Please first complete the short questionnaire on the **last page** of this problem packet in the 5 minutes you will be allotted for this, **before the exam**.
Only start working on the exam when you are told to do so. You may not be able to complete all of it. Start with the problems you find easiest and work onward from there. The problems are ordered loosely according to their difficulty. Some partial credit will be awarded.

You may use the space on this examination for scratch work. However, **all work to be graded must be in the exam workbook given to you. Box all answers, and try to work as clearly and neatly as possible.**

Distribution of Marks

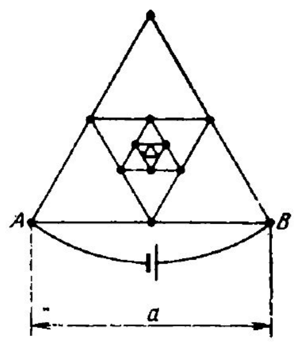
Question	Points	Score
1	10	
2	15	
3	15	
4	25	
5	10	
Total:	75	

1. Two identical cylinders are released at the same time from the top of two nearly identical ramps. The two ramps have the same shape, but because the first ramp has a special surface, the first cylinder will roll with slipping for a portion of its journey down the ramp. If the second cylinder rolls without slipping the whole time:

   (a) (5 points) Which cylinder has greater linear velocity at the bottom of the ramp?

   (b) (5 points) Explain why.

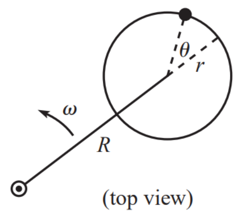
2. Problem 3.27 in S.S. Krotov's *Aptitude Test: Problems in Physics.* Consider an arrangement of homogeneous wire into an infinite sequence of nested equilateral triangles whose perimeters are halved at each step. Let ρ be the resistance per unit length of the wire and λ the length of one side of the initial triangle. Define $R := a\rho$.



   (a) (7 points) Draw an equivalent circuit consisting of finitely many components with resistors R, $R/2$, and R_{AB} where R_{AB} is the total resistance between the points A and B shown. Carefully explain why this is in fact an equivalent circuit to that above.

   (b) (8 points) Find R_{AB} in terms of R.

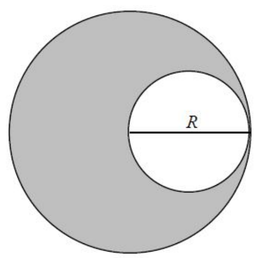
3. Problem 6.12 in David Morin's *Introduction to Classical Mechanics*. A bead slides along a frictionless, massless hoop of radius r that is being rotated in the horizontal plane at constant angular speed ω such that the center of the hoop travels in a circle of radius R.



(top view)

   (a) (5 points) Write down the Lagrangian $\mathcal{L}$ of the system
   (b) (5 points) Find the equation of motion in terms of the angle θ shown.
   (c) (5 points) Find the frequency of small oscillations about the stable equilibrium point and discuss the limiting cases $R \ll r$; $R \gg r$ and $R \approx r$.

4. Consider a simple physical pendulum that consists of a rod of length l and mass m_r along with a solid uniform disk of radius R and mass m_d attached to one end of the rod so that its center corresponds with the end of the rod. Assume initially that the k is fixed so that it doesn't rotate.
   (a) (3 points) What is the moment of inertia of the combined system about the pivot point?
   (b) (7 points) Let θ denote the displacement of the system from the vertical. Using either torque analysis or the Lagrangian formulism, find the equation of motion of the system in terms of θ.
   (c) (3 points) What is the frequency of small oscillations about the vertical?
   (d) (5 points) Assume now that the disk is free to rotate about its center. In this case, how does the frequency of small oscillations compare to your answer in the previous part?
   (e) (7 points) This part is entirely independent of the other parts of the question. Consider a solid uniform disk of mask M and radius R from which a smaller disk of radius $R/2$ has been removed as shown. Find the moment of inertia of the resultant object about its center of mass.



5. This question will not involve any math and is designed to be open-ended. Though we will be very generous with the grading, please finish the other parts of the exam before thinking about this problem. This problem is based off of the introduction to Hugh Everett III's *The Many-Worlds Interpretation of Quantum Mechanics.*

   In an isolated room in deep space, a person A is about to make a measurement of a system S. Suppose that although person A knows the possible results of the measurement, he has no prior knowledge about the state of the system S, i.e. from the perspective of person A, the outcome of the measurement is entirely random. Meanwhile, just outside the room, there is a person B with knowledge of all the possible states of the combined system consisting of A, S, and the measuring device. However, like person A, person B is unable to say what the result of the measurement will be. To her, the outcome of the experiment is also entirely random. Given the knowledge she does have, however, person B is able to find all possible states of the combined system a week later, so she makes this computation and leaves to allow person A to perform the experiment undisturbed and write down the result. Person A is very happy with the measurement, so he decides to throw a party and celebrate before person B returns a week later to see how the experiment went. Person B, being an astute quantum theorist, knows that since she had no way of knowing how the experiment would go before she returned, person A's entire perception and experience of the past week had no objective existence before she saw the measurement. In other words, up until the point that person B saw the measurement, person A's reality, the party, his emotions, didn't actually exist in any definite sense.

   (a) (10 points) To avoid this nihilistic conclusion, try to find a weakness in the argument above by casting doubt on the legitimacy of one its assumption. For instance, you could say that there is actually only one valid observer in the entire Universe, so it doesn't make sense to talk about person B's observations in the first place. To help the graders, please use the following format to answer:

      1. Point out an assumption that you would like to challenge.
      2. Offer an alternative to this assumption.
      3. Weigh the merits of this alternative.

      Making an honest attempt at 1 and 2 will earn you 6 points, and making an honest attempt at 1,2, and 3 will earn you 8 points. Entirely valid reasoning that addresses 1,2, and 3 will receive all 10 points.

This page is intentionally left blank as a space for scratch work. As a reminder, **all work to be graded must be in the exam booklet given to you.**

1 Short optional questionnaire: complete before the exam!

For internal research purposes, to help us better understand students' performance trends in aggregate form, we are including these questions to be completed within a designated period of 5 minutes **before** the exam. **Please tear out this page and hand it to the proctors separately when they collect it.** Answers to these questions will only be tied to scores in an anonymous manner, using your Competitor ID (Exam Code) number: please write it below.

Competitor ID (Exam Code): ______________________________
Initials: ____

(a) Have you prepared for this competition in any way beyond your regular coursework? If so, how? (e.g. went through specific practice problems, participated in another Olympiad/competition, etc.) Please name any relevant resources.

(b) Academically, do you consider yourself a natural introvert (you like working individually more) or a natural extrovert (you prefer working in groups or teams)?

1. I prefer working individually
2. I prefer working in a team
3. It doesn't matter to me

(c) Do you think that becoming part of an online global community of like-minded physics enthusiasts might be helpful for you in the future? If so, would you be interested in receiving more information through email about this opportunity later?

1. Yes, my email is: ____
2. No, not interested