Physics Unlimited Premier Competition

November 18, 2018

Directions

The test consists of four problems, which you will be given 1.5 hours to complete. No collaboration is allowed.

Some useful test-taking hints:

- You may not be able to complete every problem. Keep moving; do what you know first.

- Make your answer clear by circling it.

- Use symbols rather than numbers wherever possible and check units.

- Wherever possible, check whether an answer or intermediate result makes sense before moving on.

- If you get stuck on an early part of a problem, check whether you can still do the later parts; some may be independent and doable.

- If you get stuck on an early part of a problem, and a later part depends on it, clearly define a symbol for the unknown answer and use it in later parts. However, keep in mind that we often give multiple parts to guide you through a problem.

To get full credit you need to show your work! Partial credit will also be awarded at the judges' discretion.

**Good Luck!**

1   A Sliding Ladder

A ladder of mass m and total length $2l$ is initially held at rest as shown in the diagram below. The ladder is gently allowed to rotate and slide. Assume there is no such thing as friction throughout the exam.
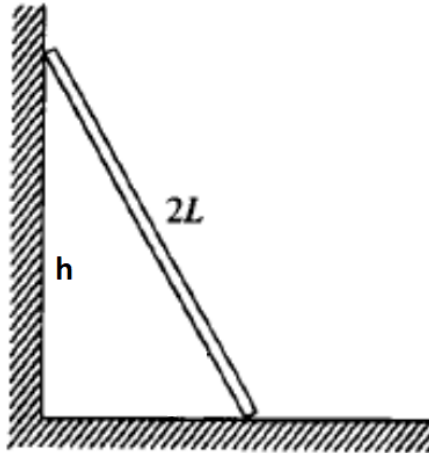
a) Provide an accurate force diagram depicting the situation immediately after the ladder has been released from rest.

b) Identify and label on this force diagram the point about which you will consider analysis of the torques.

c) Write down all relevant equations of motion. [*Hint: There is both rotational and translational motion.*]

d) In terms of the initial value h, find the height at which the top of the ladder loses contact with the wall.

2 The Hohmann Transfer

Let R denote the radius of the earth. A satellite of mass m is initially found in a circular orbit of the earth a distance $2R$ from its center, as shown below. The parts that follow will ask a few questions about the nature of its orbit.
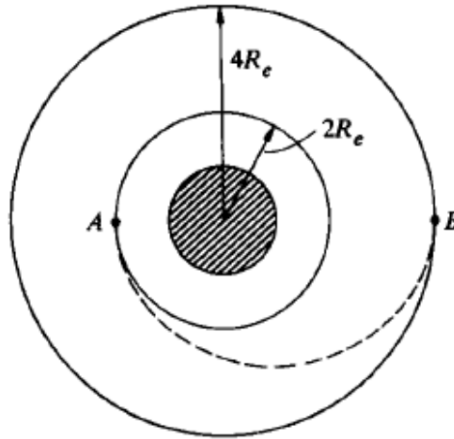
a) What would be the minimum amount of energy needed to take the satellite to another circular orbit a distance $4R$ from the center of the earth? [*Hint: Write down an equation for the total energy of the satellite, and then consider the particular situation.*]

b) In light of the hint, derive an expression for the energy of a satellite of mass m in an elliptic orbit of semi-major axis length a about a much larger object of mass M for $a = \frac{r_{\max} + r_{\min}}{2}$ where $r_{\max}$ and $r_{\min}$ represent the satellite's maximum and minimum distances from the center of the larger body, respectively. [*Hint: You will need to apply conservation of angular momentum at some point.*]

c) The Hohmann transfer utilizes a semi-elliptical orbit to efficiently move a satellite from one circular orbit to another farther away from its center. Utilizing the result of part b), how must the satellite's speed change at the points A and B to accomplish the Hohmann transfer, which is shown on the diagram given? [*Hint: Write down equations for the total energy at the points A and B.*] If you were unable to find the answer to part b), define a dimensionally correct expression for the energy of a satellite in an elliptic orbit and proceed using that expression.

d) Check the result of part c) against your answer in part a). Are they consistent?

3 Macroscopic Quantities of Gases

Suppose you are the principal investigator of a lab and one of your lab assistants comes to present his new findings. He shows you the following table:

Experiment	Volume (m^3)	Pressure (Pa)	Temperature (K)
1	100	1	200
2	150	1	300
3	50	2	200
4	100	2	400

He then tells you that all these measurements where taken with the same gas, preventing any kind of leakage.

a) What hypothesis could you formulate that mathematically summarizes all the correlations in the table?

b) What new experiments could you suggest to your lab assistant in order to verify your previous hypothesis? Describe not only the quantities being compared in the experiment, but also the experimental setup.

4 The Momentum of Electromagnetic Fields

Contrary to intuition, electromagnetic fields convey both linear and angular momentum despite having no physical composition. To help convince you of this, the parts that follow will motivate such an assertion through the use of a physical example. A few useful formulae are provided below. [*Note: If you are not particularly comfortable with a bit of vector calculus and/or have not seen this material, you are strongly encouraged to first complete the other three problems before attempting this one. As a reminder, partial credit will be awarded.*]

We denote vectors with boldface. The following will be necessary to solve the problem:

- The impulse $\mathbf{J} = \Delta \mathbf{P} = \int \mathbf{F} \mathrm{d}t$.

- The linear momentum *density* stored in the electromagnetic fields $\wp = \epsilon_0 (\mathbf{E} \times \mathbf{B})$.

- The force on a current carrying wire $\mathbf{F} = \int I d\mathbf{l} \times \mathbf{B}$.

- $\mathbf{F} = q\mathbf{E}$ is the force on a charge q in an electric field $\mathbf{E}$.

- In integral form, Faraday's law of *induction* reads $\oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = - \int \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{d}\mathbf{A}$.

Consider a parallel plate capacitor with plate separation d and plates of area A. Assume the electric field $\mathbf{E}$ between the plates of this capacitor is in the $\hat{z}$-direction and that the capacitor is placed in a uniform background magnetic field $\mathbf{B}$ in the $\hat{x}$-direction.
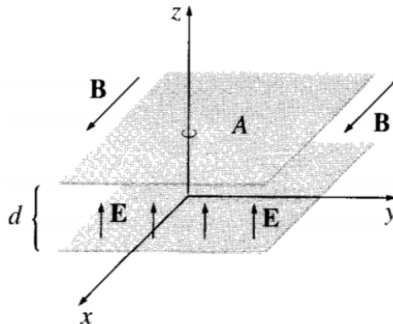
a) Find the total linear electromagnetic momentum between the plates.

b) Connect the plates with a resistive wire stretched along the z-axis. As the capacitor discharges, the current in the wire will experience a force due to the background magnetic field. Show that the total impulse delivered to the system is equal to the momentum originally stored in the electromagnetic fields.

c) Consider an alternative to the situation in part b). Slowly turn off the magnetic field and let the induced electric field exert a force on the plates. Again, show that the total impulse delivered to the system is equal to the momentum originally stored in the electromagnetic fields.