Physics Unlimited Explorer Competition

November 2017

Guidelines: The assignment consists of two sections, which you will be given 2 weeks to complete. One section is on tidal heating and the other is on relativistic electrodynamics. Collaboration only among your team members is allowed. Teams may complete both sections or choose to complete only one, as is specified in the grading explanation below. We recommend that teams set aside approximately 20+ hours to allow enough time for successful completion. Please refer to the submission explanation below for details on both formatting and the submission process.

Some useful tips and requirements:

- You may not be able to complete each problem. Do not worry, as all parts are weighted differently, and a significant amount of partial credit can be granted for reasonable explanations.
- Please make your final answers, wherever appropriate, clear by boxing them.
- Use symbols rather than numbers wherever possible and check units.
- Wherever possible, check whether an answer or intermediate result makes sense before moving on.
- If you get stuck on an early part of a problem, and a later part depends on it, clearly define a symbol for the unknown answer and use it in later parts. However, keep in mind that we often give multiple parts to guide you through a problem.
- Teams may use any information they find useful on the Internet. However, under no circumstances may they actively post content or ask questions about the exam.
- Teams may use any computational resources they might find helpful, such as Wolfram Alpha/Mathematica, Matlab, Excel, or lower level programming languages (C++, Java, Python, etc). For some sections, the use of computational resources is highly advised.
- Teams may take advantage of any published material, both printed or online. However, the requirement is that *all* student submissions with outside material must include numbered citations. We do not prefer any style of citation in particular. Students may find the following guide useful in learning when to cite sourced material: http://www.princeton.edu/pr/pub/integrity/pages/cite/.
- Piazza page: Teams are encouraged to create an account on Piazza and register in the class at the following URL: http://piazza.com/physicsu.org/fall2017/ec2017. This resource can be used by teams to ask questions about the content of the exam. Please do not post any of your solutions, partial or complete, when asking questions on Piazza, as it may be grounds for disqualification.

To get full credit you need to show your work! Partial credit will be awarded at the judges discretion. **Good luck!**

0.1 Grading

Students are encouraged to work on as much of both sections of the exam as possible. However, teams may choose to submit solutions for only one of the two sections if they desire. The two sections will be graded separately, and may not necessarily be worth the same amount of points. The award structure will be as follows:

a. Awards will be given to the four teams with the highest score in each section (an award for first place, second place, third place, and fourth place). One team can win an award for both sections, such as second place in Tidal Heating and fourth place in Relativistic Electrodynamics. Therefore, teams are encouraged to attempt solutions for both sections of the competition.

b. We will additionally award one overall award to the highest scoring team on the entire competition. A team which wins this overall award can still receive one of the top four awards for each individual section. The team that wins this overall award will most likely have completed both sections of the exam. It will be at the judges discretion to choose the overall award for the best submission.

c. Special awards will also be given for honorable mentions, the most elegant solution, and the most creative solution.

0.2 Submission

All submissions, regardless of formatting, should include a cover page listing the title of their team, the date, and signatures of all team participants. The work for each section must be submitted as one single PDF document with the .pdf extension. This means that if your team worked on two sections, you should submit two PDF files in a single e-mail message, as specified below. All other formatting decisions are delegated to the teams themselves. No one style is favored over another. That being said, we recommend that teams use a typesetting language (e.g., LATEX) or a word-processing program (e.g. Microsoft Word, Pages). Handwritten solutions are also allowed. Note: we reserve the right to refuse grading of any portion of a teams submission in the case that the writing or solution is illegible.

Teams must submit their Explorer Competition solutions by e-mailing directors@physicsu.org by 11:59 pm Eastern Standard Time (UTC-5) on Sunday, November 26, 2017. Teams will not be able to submit their solutions to the Explorer Competition at any later time, unless there is a small time extension publicly granted by the organizers. Any team member may send the team's submission, but please consider doing your best to avoid sending more than once; the last version from a team submitted before the deadline will be the submission that is graded. The title of the submission email should be formatted as "SUBMISSION - Team Name". Teams will receive confirmation once their submission has been received within at most two days. In the case of extraordinary circumstances, please contact us as soon as possible.

Thank you to the sponsors and collaborators of both the Princeton University Physics Competition and the independently run Physics Unlimited, the non-profit organization overseeing and directing the Explorer Competition beginning in 2017.

Contents

Chapter 1

Tidal Heating Section

1.1 Introduction

Sometimes, as physicists, you might encounter some strange phenomenon that you will have to explain. Most certainly, you will want the explanation to be compatible to modern physics. Of course, such an explanation must be not only qualitatively reasonable, but also quantitatively acceptable. While the former is usually trivial, the latter can quite difficult and particularly uncomfortable if there is only limited data available.

Certainly, the best way to validate your explanation would be to have some well understood functions that represent the system's physics. When given some measured data, these functions would return other data that could be checked against measurements. However, we can do so only for simple systems, such as pendulums and the orbits of celestial bodies around a very massive object. Next (and almost just as good) would be to run a simulation and show the agreement between the simulation output and the measured values (and prove that disagreement is only due to the simulation limitations). Again, we might not be so lucky - writing software capable of simulating the whole range of interactions which define the phenomenon could turn out to be exceedingly difficult. Therefore, we are only left with figuring out some simplified model by ourselves and doing some very approximate computations to get the desired results.

Such is the case of the tidal heating of Io, the innermost of Jupiter's Galilean moons. Io's surface energy flux is significantly (orders of magnitude) higher than expected from the usual processes in most planetary bodies in our Solar System. The proposed explanation is that the excess energy is due to the tidal heating of Io's interior as the satellite orbits around Jupiter.

1.2 Task

Your challenge in this section is to develop a reasonable model for Io's tidal heating, and compute the model parameters. Compare the results to others in the scientific literature, then write an article presenting your model as for publishing in a scientific journal.

Steps Required for Successful Completion of the Task

Developing your Model Points will be awarded based on how well you incorporate the following considerations into your explanation.

a. Determine how Io deforms in Jupiter's gravitational field. Make sure to use relevant approximations and explain why they can be used.

b. Compare Io with other planetary bodies in the Solar System for which we know the internal structure and give a qualitative model of Io's interior. You will want to use results given by inducted magnetic fields on Io.

c. Observe the way Io's surface changes in time under tides. Considering the height of tides on Io, find which forces are relevant in your model. Deduce that there is a very well-known class of problems that approximates Io's behavior.

d. Find the heat generated and compare the results obtained with those in scientific literature. Comment on Io's surface temperature. Discuss briefly the energy balance of Io's volcanoes.

e. Find the energy source heating Io and estimate what will happen to the Jovian moon(s) in the future.

Publishing Your Model Now that you have a working model, imagine you want to publish it in a scientific journal.

f. Write an article detailing your model, reporting the inferred parameters and comparing the results to those obtained by others.

Tips & Notes

- A simpler model is preferred to a more complex one if accuracy does not suffer appreciably.
- There is a significant lack of measured characteristics about Io, and thus some particularly useful data is inferred indirectly, with low accuracy. It is then very likely that you will need to make sensible choices about the parameters of your model.
- Correct approximations simplify the model greatly (and a working model is impossible without approximations). Make sure (and convince us) that the approximations are correct.
- The last part (writing an article) is not allocated many marks, such that it will not hamper well-developed models. We encourage you to work on it, though, as doing so will teach you valuable skills for a future in scientific research.

1.3 Post-Contest Challenge

Try simulating Io's tidal heating behavior or thinking about what this simulation would entail.

Chapter 2

Relativistic Electrodynamics Section

The subject matter of this section is relativistic electrodynamics, that is, the study of electricity and magnetism incorporating Einstein's Special Theory of Relativity . The goal of this topic is to give students a feel for the beauty of Maxwell's equations (the core equations of electricity and magnetism) when formulated in relativistic language and to give them an understanding of unification of these two seemingly different subjects into a grander electromagnetic field.

In introductory electricity and magnetism classes, students typically learn about electric fields and magnetic fields and perhaps learn about the basics of how the two are connected, but such courses tend not to give the student an understanding of the deep connection between these two fundamental subjects. In particular, students miss out on the fact that electricity and magnetism are not two different phenomenon, but rather part of the same overarching mathematical structure (the electromagnetic field tensor, which will be introduced in this section). In fact, what one calls magnetism can be viewed as manifestation of a moving electric field and vice versa. Furthermore, while students are introduced to Maxwell's equations, they are not shown how they can be condensed into a single fundamental equation with the language of four vectors. This section teach students how to do this, offering derivations and problems along the way. Our medium will be relativity, so that will be introduced first. Then we will get into reformulating electrodynamics.

Learning goals of this topic: As stated above, the goal of this topic is to give students an appreciation for the beauty of the relativistic formulation of electricity and magnetism and an understanding of the deep relationship between the subjects.

Topic format: This document consists of explanatory sections with helpful exercises and questions interspersed. Much of the grading will be based on sections that ask you to explain or interpret results in your own words. We are looking to see how well you understand the subject and are less concerned with minor errors in completing exercises.

Some sections contain no questions. We would encourage you not to skip reading these parts, as all sections will be beneficial for your understanding. If your first read-through leaves you perplexed, baffled, or utterly nonplussed, don't be discouraged: these topics take time to absorb!

2.1 Basics of Special Relativity

2.1.1 Conceptual Basics of Special Relativity

Perhaps the most exciting aspect of learning special relativity is having your physical intuition turned on its head and sent through a meat grinder. Please refer to this webpage[1] to get a conceptual background for special relativity, then answer the questions below. You may skip the sections on World Lines and Light Cones.

Problem: The Barn Paradox

This is a classic problem in special relativity; it will be up to you, however, to determine whether or not it is truly deserving of the label “paradox”.

In Fig. 2.1, a runner is carrying a pole that is slightly longer than the length of a barn. The barn in question has two doors; one in the front and one in the back. The runner is capable of running at speeds comparable to the speed of light, and wants to know if she can fit the length of the pole inside the barn in a single instant; to test this, a farmer standing outside the barn will close both of the doors at the same time, and then open them again immediately after (you may assume the doors take only an infinitesimal length of time to open and close, and remain closed for a negligible amount of time). Is there any way the pole will fit inside the barn? Describe the events of the runner entering the barn, the doors closing, and the runner leaving the barn from the point of view of the runner and of the farmer. Is this really a “paradox”? Why or why not?

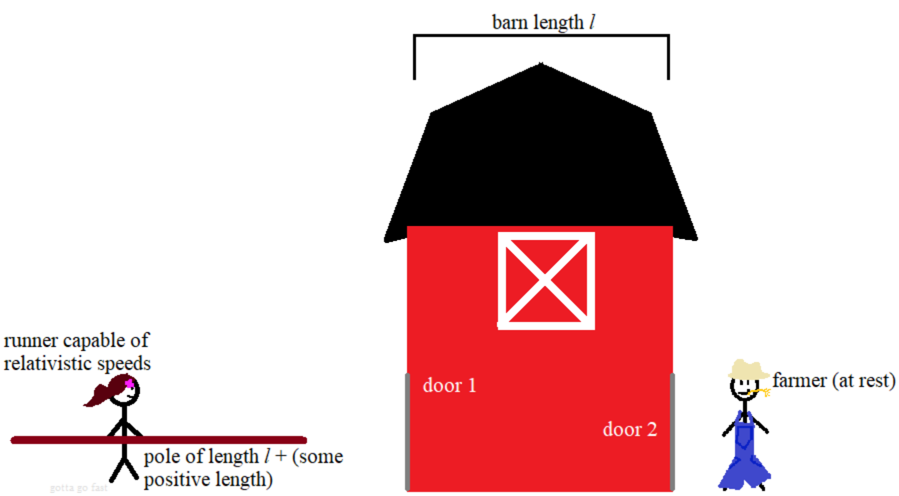

Figure 2.1: The Pole in the Barn Paradox

[1] Copy and paste http://abyss.uoregon.edu/~js/21st_century_science/lectures/lec06.html into your browser if the link doesn't work.

2.1.2 Introduction to Four Vectors

Up until now, you have likely done most or all of your physics with three-vectors, collections of three numbers which represent some physical quantity, such as position, momentum, force, some field, etc. In special relativity, it will be more useful to extend this concept to that of a four-vector.

To introduce four-vectors, consider the collection of coordinates $dx^\mu = (dx^0, dx^1, dx^2, dx^3) = (cdt, dx, dy, dz)$[2] Consider two frames–S and S' (see Fig.2.2). S is stationary and S' moves in the positive x-direction with velocity v relative to S. The spacetime coordinates in S are given by (cdt, dx, dy, dz) and those in S' by (cdt', dx', dy', dz').

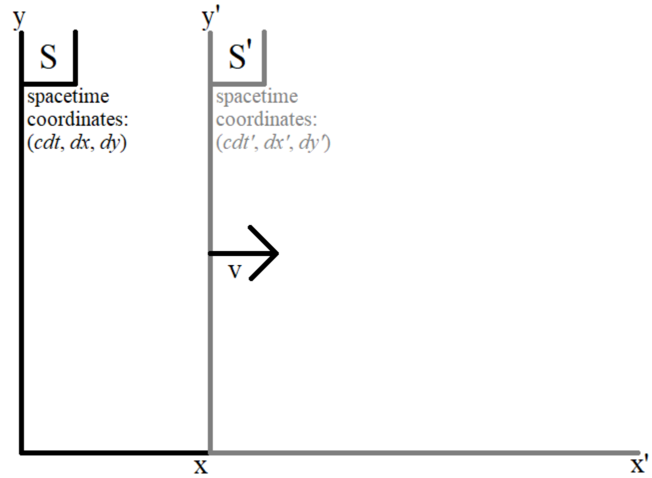

Figure 2.2: Lorentz Coordinate Transformations

According to special relativity, the coordinates are related by the Lorentz transformation[3]

$$cdt' = \gamma(cdt - \beta dx)$$

$$dx' = \gamma(dx - \beta cdt)$$

$$dy' = dy$$

$$dz' = dz,$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ and $\beta = \frac{v}{c}$.

Or, in matrix form,

$$\begin{pmatrix} cdt' \\ dx' \\ dy' \\ dz' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix}.$$

A transformation between inertial frames such as this is known as a boost. Unless otherwise noted, the remainder of the problems in this portion of the exam will refer to boosts.[4]

$dx^\mu = (cdt, dx, dy, dz)$, and any other collection of four-coordinates which obey the Lorentz transformation under a boost, are known as four-vectors.

[2]This superscript notation is new. Essentially, we are saying that dx^μ represents the whole vector, since you can reproduce the entire vector by plugging in each of the different values of μ (from 0-3).

[3]These can be derived directly from Einstein's postulates and an assumption of linearity; however it will be more convenient to simply start with them for our purposes. Furthermore, the velocity can be in any direction. We have used the x-direction for simplicity.

[4]Lorentz transformations also include regular spatial rotations. And a four vector can also be subject to translation. However, these will not be explored during this section of the exam.

2.1.3 Einstein Summation

Einstein summation is typically used for notational brevity; instead of writing out a sum each time we want to refer to a quantity, we can simply write out two terms with indices. When you see two quantities multiplied together, multiply the ith term from each quantity and sum over all indices.
In other words,

$$a_\mu b^\mu = \sum_{i=\mu}^{n} a_\mu b^\mu;$$

here, μ represents the index and both quantities are four-vectors. Notice one index is a superscript and the other is a subscript; the quantity with a superscript has vertical vector components, and the quantity with the subscript has horizontal vector components.[5] For the purposes of this section, n is equal to 4; this term represents the dimension of the space.

2.1.4 The Spacetime Interval

Consider the collection of coordinates dx^μ in the frame S from section 1.1. We have coordinates (cdt, dx, dy, dz), and since the index is noted as a superscript, we know that this is a column vector (called a "contravariant vector"). If we wanted to change our contravariant vector to a row vector (covariant vector), we simply make the first component negative, and the vector becomes dx_μ with coordinates $(-cdt, dx, dy, dz)$.[6]

We can compute the spacetime interval (the squared length of a differential amount of spacetime) by

$$ds^2 = dx_\mu dx^\mu.$$

This is a four-dimensional dot product, and ds is the norm of the displacement four-vector.

Problem: Invariance of the Spacetime Interval

Expand the above product and show that it is invariant under Lorentz transformations.

Problem: Time Dilation

Using the spacetime interval, compute the relation between (differential) time in the unprimed frame and the primed frame.[7] This is known as time dilation. Specifically, what does this imply in terms of how quickly time passes between two frames? Give a real-world example of time dilation in action (you may greatly exaggerate the effects).

Problem: Length Contraction

Compute the relation between length in the unprimed and primed frames. This is the other half of the

[5] This may look a little familiar if you've had a lot of practice with the dot product in more mathematical applications; if not, you needn't worry too much about this detail.

[6] In fact, we are changing from a contravariant to a covariant vector via the metric of the space (the Minkowski metric in special relativity). However, we will not go into those details in this portion of the exam.

[7] The primed frame is known as the proper frame. A particle which moves with a certain velocity in the unprimed frame is stationary in this frame.

special relativity puzzle, known as length contraction. Give an example of the relationship between time dilation and length contraction; in other words, give an example of a scenario in which different observers will experience one or the other of these effects. Be sure to explain your reasoning. *(Hint: you may not use the barn problem from above, but it can serve as a good starting point to come up with your own scenario.)*

Problem: Relativity and Rotations

The boost transformation law is typically derived by shifting between inertial frames and using the postulates of relativity, as well as an assumption of linearity. However, one can also derive them by a rotation. In 3-space, a rotation by angle θ about the z-axis transforms the coordinates as

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$z' = z$$

Note that this rotation preserves length $ds = \sqrt{x^2 + y^2 + z^2}$.

Using the idea that the Lorentz transformation must preserve the spacetime interval, choose a system of four coordinates and use the rotation law above (rotation of two axes about a third) to derive the Lorentz transformation. *(Hint: use the relation between γ and β to define the angle of rotation.)*

2.1.5 Mechanics in the language of four-vectors

Now that you have been introduced to four-vectors, let's formulate all of mechanics in terms of them.

The four-velocity is given by: $u^\mu \equiv \frac{dx^\mu}{d\tau}$.
Here $d\tau$ denotes proper time, which is the time as measured by an observer moving in S' (the moving frame). In short, $d\tau = dt'$.

The four-momentum by: $p^\mu = m_0 u^\mu$, where m_0 is the rest mass (mass in a stationary frame).

And the four-force by: $F^\mu \equiv \frac{dp^\mu}{d\tau}$.

Problem: Four-Velocity

a) Why does one have to differentiate with respect to proper time? What happens if you don't?

b) Derive the Einstein velocity addition laws from the Lorentz transformation for the four-velocity. These laws are:

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}.$$

Problem: Invariance of Energy and Momentum

The momentum four-vector is given by $p = (\frac{E}{c}, p_x, p_y, p_z)$. Use this information to derive the relationship between energy and momentum.

Problem: Four-Acceleration

Prove that the dot product of four-acceleration and four-velocity is zero for any arbitrary mass m.

2.2 Relativistic Electrodynamics and Tensors

2.2.1 Four-dimensional Calculus

One can extend the gradient of three dimensions to four as

$$\partial_\mu = (\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3})$$

.

The divergence can then be defined as the (four-dimensional) dot product of this gradient with an arbitrary four-vector.

A further operator, the d'Alembertian, may be defined as follows: $\Box^2 \equiv \frac{1}{c^2}\frac{\partial}{\partial t^2} - \nabla^2$, where ∇^2 is the ordinary Laplacian.

2.2.2 Four-Current

Using the four-velocity, we can define a current density[8] four-vector as

$$j^\mu \equiv \rho_0 u^\mu.$$

Problem: The Continuity Equation

a) Prove, that in 3 dimensions,

$$\vec{\nabla} \cdot \vec{j} = \frac{-\partial \rho}{\partial t}$$

. This is known as the Continuity Equation. *(Hint: consider a small sphere with charge flowing outward and use the fact that charge is conserved.)*

b) Now show that, with four-vectors, you can express this neatly as

$$\partial_\mu j^\mu = 0.$$

What does this mean in terms of the four-dimensional divergence?

[8] In 3-dimensions, current density is defined as $\vec{j} = \rho\vec{u}$.

2.2.3 Four-Potential

In terms of 3-vectors, Maxwell's equations (the core equations of electricity and magnetism) read

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$c^2 \vec{\nabla} \times \vec{B} = \frac{\vec{j}}{\varepsilon_0} + \frac{\partial \vec{E}}{\partial t}$$

Problem: Maxwell's Equations in Terms of the Potentials

a) From these equations, show that $\vec{B}$ and $\vec{E}$ may be written as

$$\vec{B} = \vec{\nabla} \times \vec{A}$$
$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t},$$

where $\vec{A}$ is known as the vector potential and ϕ is known as the scalar potential. Specifically, which of Maxwell's equations did you use to attain these equations and which properties of vector calculus were used?

b) Using the conditions from a), show that one may write Maxwell's equations as

$$\Box^2 (\frac{\phi}{c}) = \frac{c\rho}{\mu_0}$$
$$\Box^2 \vec{A} = \frac{\vec{j}}{\mu_0}$$

You may find the relation $\vec{\nabla} \times (\vec{\nabla} \times \vec{C}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{C}) - \nabla^2 \vec{C}$, where $\vec{C}$ is some vector, useful.

Note: In this derivation, it will be necessary to exercise something known as gauge freedom, which means that you will have to choose a value for $\vec{\nabla} \cdot \vec{A}$. This choice can be anything, so long as it helps you to attain the form of the equations given above. Why did you make this choice? How did you know it was the proper choice to simplify the equations?

c) Show that $A^\mu = (\frac{\phi}{c}, \vec{A})$ is a four-vector by

i. showing that $\Box^2$ is Lorentz invariant. Explain why is this condition sufficient show that A^μ is a four-vector.

ii. by showing dV/r is Lorentz invariant. Why is this condition sufficient? It may help to recall that $\phi = \int \frac{\rho dV}{r}$ and $\vec{A} = \int \frac{\vec{j} dV}{r}$.

d) Using the fact that A^μ is a four-vector, show that all of Maxwell's equations may be written as

$$\Box^2 A^\mu = \frac{j^\mu}{\mu_0}.$$

Be sure to carefully explain your logic in getting from the equations of part b) to here.

Problem: Forces in Different Frames

An electric force F_e acts on a particle of mass m and charge $q+$ that is moving at a velocity v in the unprimed frame. Find F', the same force in the particle's rest frame.

Problem: Particles in a Wire

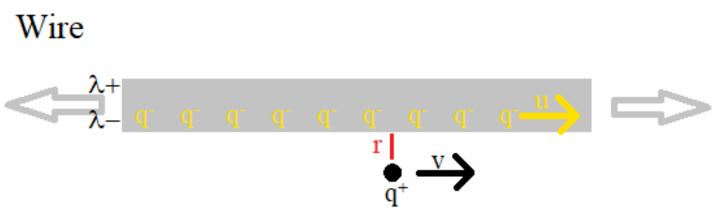

Figure 2.3: Particles in a Wire

Refer to the Fig. 2.3 above. In the lab frame, negative particles are moving in a wire at a speed u. The positive particles are stationary. The positive particles have a charge density $\lambda+$, and the negative particles have a charge density $\lambda-$.
There is another particle of charge $q+$ a distance r away from the wire, moving in the same direction as the negative particles in the wire at a speed v.

a) Find the total force on the particle of charge $q+$ in the lab frame.

b) Find the total force on the same particle in its own rest frame. What does this say about the nature of electric and magnetic forces? Use your answers from the previous parts of this question in your explanation.

2.2.4 The Electromagnetic Field Tensor

a) Using the equations from 3.3.1a, write the components of $\vec{E}$ and $\vec{B}$ in terms of the components of A_μ and ∂_μ, using $A_\mu = (A_0, A_1, A_2, A_3) = (\frac{\phi}{c}, A_x, A_y, A_z)$ and $\partial_\mu = (\partial_0, \partial_1, \partial_2, \partial_3) = (\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$.

b) Using these examples, write a general expression for a field component in terms of A_μ and ∂_μ, which will be a function of two indices, μ and ν. Call this $T_{\mu\nu}$. This is known as the electromagnetic field tensor.

c) What happens when $\mu = \nu$ and when the two indices are flipped? How many entries does this account for? How many entries does $T_{\mu\nu}$ have in total?

d) Now write down $T_{\mu\nu}$ as a matrix.

2.2.5 The Transformations of the Fields

The reason the matrix above is so crucial is because of how it transforms. It is a rank 2 tensor and so transforms in an understandable way under a boost. Applying this transformation gives us how the electric and magnetic fields change under a boost.

a) The Lorentz transformation for the electric and magnetic fields is given in tensor language as

$$T'_{\mu\nu} = \Lambda T_{\mu\nu} \Lambda^t,$$

where $T_{\mu\nu}$ is the field tensor, Λ denotes the Lorentz transformation matrix from earlier and t denotes transpose. From this relation, show how the electric and magnetic fields transform under a boost.

More generally, the fields transform as

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel}$$
$$\vec{B}'_{\parallel} = \vec{B}_{\parallel}$$
$$\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \vec{v} \times \vec{B})$$
$$\vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \frac{1}{c^2}\vec{v} \times \vec{E}).$$ [9]

b) Using Coulomb's law for the electrostatic field of a point charge, $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^3}\vec{r}$, and the transformation laws above, derive the law for the magnetic field produced by a moving charge. What does this tell you about the relation between electric and magnetic fields? This is known as the Biot-Savart law for point charges. (It may easily be extended to compute the magnetic field of currents.)

c) Now, using the Biot-Savart law for point charges, and the field transformations above, derive Coulomb's law (take the limit of small velocities). What does this tell you about the relation between electric and magnetic fields?

2.2.6 Field Transformation Problems

Problem: Moving Solenoid

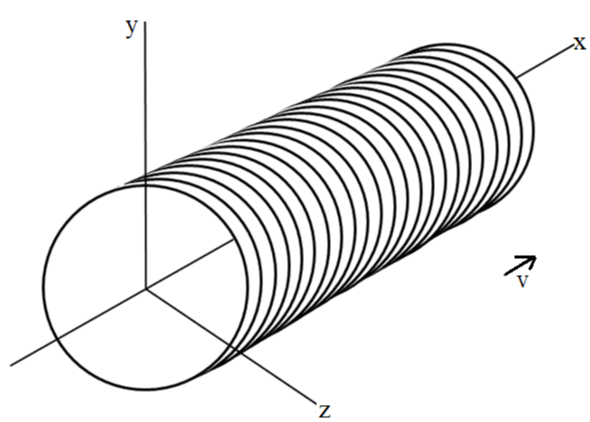

Figure 2.4: Solenoid with N Turns

A long solenoid with N turns is positioned in frame F, as shown in Fig. 2.4 above. If the solenoid moves in the positive x direction at a velocity v, find the electric and magnetic fields inside the solenoid in frame F and in the solenoid's rest frame.

[9] The parallel and perpendicular signs are in reference to direction of motion; i.e. if a particle is moving parallel to electric field lines, use the equations for $\vec{E}'_{\parallel}$.

Correction for Maxwell's Equations

We have shown that one of Maxwells equations is $c^2\vec{\nabla} \times \vec{B} = \frac{\vec{j}}{\varepsilon_0} + \frac{\partial \vec{E}}{\partial t}$. You may have also seen it written (incorrectly) as $\vec{\nabla} \times \vec{B} = uJ$. This is a simplified version of the equation that is sufficient for a lot of uses of the equation. Provide an example in which the $\frac{\partial \vec{E}}{\partial t}$ component is necessary in order to complete the problem.