Physics Unlimited Explorer Competition

November 2017

ASSIGNMENT

TTHQ TEAM

FROM: VIETNAM

Tran Xuan Tung
Txtung1410@gmail.com

Do Son Tung
tungds@gmail.com

Nguyen To Vinh Huy
nguyentovinhhuy@gmail.com

Vu Nhat Quang
Vunganha2006@gmail.com

Hanoi, 3rd December 2017

II. Relativistic Electrodynamics

a. Basics of Special Relativity

i. The Barn Paradox

Consider the reference frame of the farmer, who is standing still relative to the barn. As the runner runs at a speed comparable to the speed of light, the pole carried by her experience length contraction according to the equation:

$$L = \frac{L_0}{\gamma} \text{ with } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

With enough speed, the length of the pole would be less than or equal to l, so it can fit inside the barn at, at least, one moment. This means, to the farmer's perspective, the pole wouldn't crash into the doors.

Now, consider the reference frame of the pole, the barn would be moving toward the pole while the pole stands still. In that situation, the barn would experience length contraction, and it would be shorter than the pole. Therefore, sthe pole wouldn't fit inside the barn, which contradicts what the farmer observes. This is why this problem is paradoxical. However, time is relative, and the doors don't close simultaneously in this reference frame. To the runner's perspective, the pole's head enters the barn first while the tail is outside the barn. At that moment, the backdoor of the barn closes and re-opens immediately. After that, the head of the pole exits, and the tail enters the barn. This time, the back door closes and re-opens. Therefore, to the runner's perspective, the pole wouldn't crash into the doors of the barn.

ii. Lorentz transformation

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

$$cdt' = \gamma(cdt - \beta dx)$$

$$dx' = \gamma(dx - \beta cdt)$$

$$dy = dy'$$

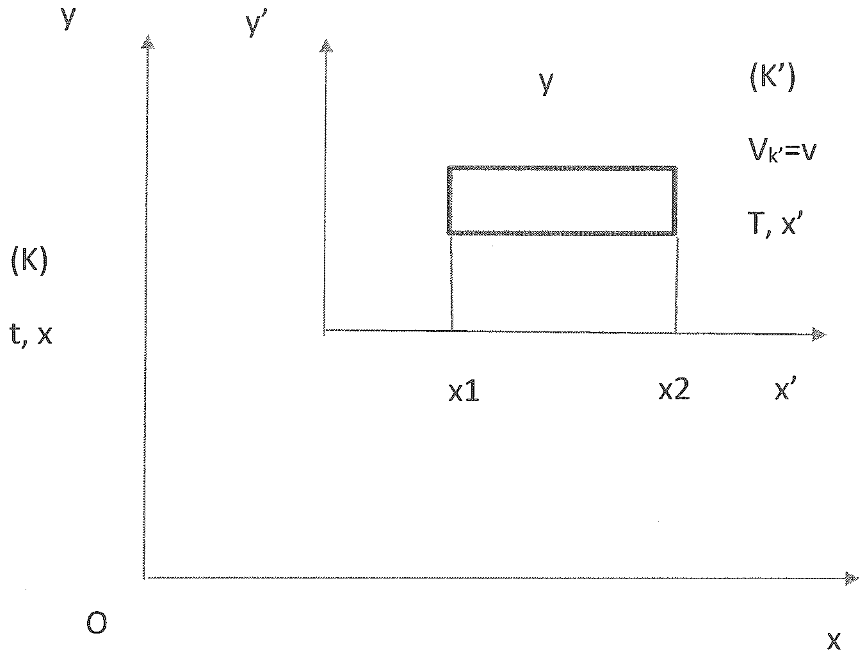
$$dz = dz'$$

$$\Rightarrow c^2dt'^2 - dx'^2 - dy'^2 - dz'^2 = \frac{(cdt-\beta dx)^2-(dx-\beta cdt)^2}{1-\beta^2} - dy^2 - dz^2$$

$$= \frac{c^2dt^2-2c\beta dt\,dx+\beta^2dx^2-dx^2+2c\beta dt\,dx-\beta^2c^2dt^2}{1-\beta^2} - dy^2 - dz^2$$

$$= c^2dt^2 - dx^2 - dy^2 - dz^2$$

$$\Rightarrow ds^2 = ds'^2$$



$x'_2 - x'_1 = L$

By Lorenz transformation

$x_1 = \gamma\,(x'_1 + c\tau)$

$x_2 = \gamma\,(x'_2 + c\tau)$

$\Rightarrow \quad x_2 - x_1 = \gamma\,(x'_2 - x'_1) = \gamma L$

Again

$C\,t_1 = \gamma\,(c\tau_1 + x)$

$Ct_2 = \gamma\,(c\tau_2 + x)$

$\Rightarrow$

$t_2 - t_1 = \gamma\,(\tau_2 - c\tau_1)$

$t = \gamma\tau$

b) Relativistic Electrodynamics and Tensors:

$$\vec{\nabla}\left(\frac{d}{d_x};\frac{d}{d_y};\frac{d}{d_z}\right)$$

$$\vec{\zeta}\left(\frac{d_x}{d_t}p;\frac{d_y}{d_t}p;\frac{d_z}{d_t}p\right)$$

$$\vec{\nabla}\vec{\zeta}=\left(\frac{d}{d_x}\frac{d_x}{d_t}\vec{p1}+\frac{d}{dy}\frac{d_y}{dt}\vec{\zeta}+\frac{d}{d_z}\frac{d_z}{d_t}\vec{\gamma}=\frac{d_p}{d_t}\right.$$

Four- mechanics

u^{u2} is invarient under Lorentz transformation

$$\Rightarrow u^u \quad \frac{cd_t}{dz};\frac{d_x}{dz};\frac{d_y}{dz};\frac{d_z}{dz}$$

If not then u^{u2} is not invariant

$$u' = \frac{d_{u'}}{d_{t'}} = \frac{\gamma(d_x - \beta c d_t)}{\gamma(d_t - \beta d_x/c)} = \frac{u_x - Vdt}{d_t - \beta d_x/c} = \frac{u - V}{1 - Vu/c^2}$$

$$u = \frac{d_u}{d_t} = \frac{\gamma(d_{x'} + \beta c d_t)}{\gamma(d_{t'} + \beta d_x/c} = \frac{u + V}{1 + Vu/c^2}$$

$$pp = (E/C; p_x ; p_y ; p_z) = m_o. U = (\gamma\ m_oc ; \gamma m_o\ v)$$

$$u = (\gamma c - \gamma\ v)$$

$$m_o^2u^2 = 1/(1-v^2/c^2)\ \ m_o^2 (c^2 - v^2) = m_o^2c^2 = E^2/c^2 - (p_x^2 + p_y^2 + p_z^2) = E^2/c^2- p^2$$

- Relativity and Rotations

$x^2+y^2+z^2-c^2t^2 = \text{const}$

$x^2+y^2 = \text{const}$

$\rightarrow \begin{cases} z' = z\cosh\alpha - ct\sinh\alpha \\ ct' = ct\cosh\alpha - z\sinh\alpha \end{cases}$

$\beta = \tanh\alpha \rightarrow \gamma = \dfrac{1}{\sqrt{1-\tanh^2\alpha}} = \cosh\alpha \rightarrow \begin{cases} z' = \gamma(z-\beta ct) \\ ct' = \gamma(ct-\beta z) \end{cases}$

- Four-Acceleration

$\underline{u}^2 = c^2 = \text{const} \rightarrow \frac{1}{2}\frac{d}{dt}(\underline{u}^2) = 0 \Leftrightarrow \underline{u}.\frac{d\underline{u}}{dt} = 0 \Leftrightarrow \underline{A}.\underline{U} = 0$

Four-current

a) $I = -\dfrac{dQ}{dt} \Leftrightarrow \oint \vec{J}.\vec{ds} = -\dfrac{d}{dt}\oint \rho\, dv \Leftrightarrow \oint(\vec{\nabla}.\vec{J})\,dv = \oint -\left(\dfrac{\partial\rho}{\partial t}\right)dv$ (Kelvinstoke)

$\Leftrightarrow \vec{\nabla}.\vec{J} = -\dfrac{\partial\rho}{\partial t}$

b) In a moving reference; $\begin{cases} x = \dfrac{x_0}{\gamma} \\ y = y_0 \\ z = z_0 \end{cases} \rightarrow v = \dfrac{V_0}{\gamma} \rightarrow \rho = \dfrac{q_0}{V} = \gamma\rho_0$

$\Leftrightarrow \rho_0\dfrac{\partial\gamma}{\partial t} - \dfrac{\partial\rho}{\partial t} = 0 \Leftrightarrow \dfrac{\partial(c\gamma\rho_0)}{\partial(ct)} - \dfrac{\partial\rho}{\partial t} = 0 \Leftrightarrow \partial_0 J^0 + \vec{\nabla}.\vec{J} = 0 \Leftrightarrow \partial_\mu J^\mu = 0$

Four-Potential

a) $\vec{\nabla}\times\vec{A} = \left(\dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z}\right)\vec{i} + \left(\dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}\right)\vec{j} + \left(\dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}\right)\vec{k}$

$\rightarrow \vec{\nabla}.(\vec{\nabla}\times\vec{A}) = \left(\dfrac{\partial A_z}{\partial x\partial y} - \dfrac{\partial A_y}{\partial x\partial z}\right) + \left(\dfrac{\partial A_x}{\partial z\partial y} + \dfrac{\partial A_z}{\partial x\partial y}\right) + \left(\dfrac{\partial A_y}{\partial x\partial z} - \dfrac{\partial A_x}{\partial y\partial z}\right) = 0 = \vec{\nabla}.\vec{B}$

$\rightarrow \vec{B} = \vec{\nabla}\times\vec{A}$

$\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} = \dfrac{-\partial}{\partial t}(\vec{\nabla}\times\vec{A}) = \vec{\nabla}\times\left(\dfrac{-\partial\vec{A}}{\partial t}\right)$

$\vec{\nabla}\times(\vec{\nabla}\phi) = \vec{\nabla}\left(\dfrac{\partial\phi}{\partial x};\dfrac{\partial\phi}{\partial y};\dfrac{\partial\phi}{\partial z}\right) = \left[\dfrac{\partial}{\partial y}\left(\dfrac{\partial\phi}{\partial z}\right) - \dfrac{\partial}{\partial z}\left(\dfrac{\partial\phi}{\partial y}\right)\right]\vec{i} + \ldots$ (Because $\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial y}\right)$)

$= 0$

$\rightarrow \vec{E} = -\vec{\nabla}\phi - \dfrac{\partial\vec{A}}{\partial t}$

$$\Box^2 \vec{A} = \mu_0 \vec{J} - \frac{1}{c^2}\vec{\nabla}\left(\frac{\partial \phi}{\partial t}\right) - \vec{\nabla}(\vec{\nabla}.\vec{A})$$

$$\Box^2 \phi = \frac{\rho}{\varepsilon_0} + \frac{\partial}{\partial t}(\vec{\nabla}.\vec{A}) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi$$

$$\Leftrightarrow \begin{cases} \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi + \frac{\partial}{\partial t}(\vec{\nabla}.\vec{A}) = 0 \Leftrightarrow \frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial \phi}{\partial t} + \vec{\nabla}.\vec{A}\right) = 0 \\ \vec{\nabla}\left(\frac{1}{c^2}\frac{\partial \phi}{\partial t} + \vec{\nabla}.\vec{A}\right) = 0 \end{cases}$$

$$\Leftrightarrow \vec{\nabla}.\vec{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = \text{const}$$

$$\rightarrow \Box^2 \vec{A} = \mu_0 \vec{J}$$

$$\Box^2\left(\frac{\phi}{c}\right) = \frac{c\rho}{\mu_0}$$

c) We have $\partial_\mu A^\mu = \frac{\partial}{c\partial t}\left(\frac{\phi}{c}\right) + \vec{\nabla}.\vec{A} = \text{const}$

$\rightarrow A^\mu$ is a four-vector

$$\left.\begin{array}{l} \Box^2 \vec{A} = \mu_0 \vec{J} \\ \Box^2 \frac{\phi}{c} = \frac{\rho}{c\varepsilon_0} = \mu_0 c \rho_0 = \mu_0 J^0 \end{array}\right\} \rightarrow \Box^2 A^\mu = \mu_0 J^\mu$$