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Tidal Heating of Io

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Abstract

Tidal heating of Io, the innermost Jovian moon, has become a topic that is visited regularly by physicists. Some preferred to look at solid aspect of Io's crust as a primary reason of heating [1], and others preferred the route of taking into account the viscosity of the magma covering Io as the main cause [2]. Although these studies took different paths regarding the root of tidal heating, both of their models are complex to a degree that it is hard to gain a cohesive understanding of the phenomena. In this paper, we try to get a relatively accurate result for the power output of the tidal heating of Io by a simpler model, but still keeping some of the ideas propesed in the papers cited.

I. Introduction

The Jovian moon Io is the only planetary body in the Solar System that is thought to have strong tidal action (by Jupiter's gravitational field) as the primary internal heat source driving its active silicate volcanism [2]. From experimental data we see that Io mainly interacts with Jupiter and Europa. Io traces an orbit with $\epsilon = 0.0041$ eccentricity around Jupiter, and its 2:1 orbital resonance with Europa causes Io to maintain this eccentricity which would otherwise tend to zero as a result of tidal dissipation. In this paper it will be assumed that Io's internal heat production is dominated by tidal dissipation [3]. This heat production mechanism has garnered attention since it is different from the mechanism that governs the Earth - radiogenic heat. Following the study [1] that initiated the inquiry for the correct physical model regarding the nature of phenomena, many solid-body and fluid-body dissipation models [5] [6] were developed [2]. However, the complexity of these models and copious calculations in them were not commensurate enough with the accuracy of the results obtained by utilizing the models. Therefore, we devise a simpler model, differing from the current understanding of the phenomena, in order to provide a qualitative explanation which can be used to derive a result that is in the range of the findings in the papers cited.

II. Ideal Fluid In Gravitational Field

To get an approximation how Io deforms under Jupiter's gravitation, lets assume Io to be an ideal fluid, which is far from reality, but it won't make a huge difference once we add the solid core inside the ideal fluid after getting an idea how the boundary of Io oscillates due to its eccentricity. For a static ideal fluid the surface must be equipotent, so we get the equation

$$\frac{\gamma M_j}{R(\theta)} + \frac{\gamma M_i o}{r(\theta)} + \frac{\omega^2 R^2(\theta)}{2} = constant \qquad (1)$$

where $\vec{R}(\theta) = \vec{R}_{io} + \vec{r}(\theta)$, $\vec{R}_{io}$ is the vector from Jupiter to center of Io, $\vec{r}_\theta$ is the vector from center of Io to surface, and θ is the angle between $\vec{R}_{io}$ and $\vec{r}(\theta)$. Also M_j and M_{io} are the masses of Jupiter and Io respectively. For the angular velocity of Io and $R(\theta)$, we get with some approximation

$$\begin{cases} \omega = \dfrac{\gamma M_j}{R_{io}^2} \\ R(\theta) = R_{io}^{-1}\left(1 - \dfrac{r^2}{2R_{io}^2}\right)\left(1 + \dfrac{R_{io} r cos\theta}{R_{io}^2 + r^2}\right) \end{cases} \qquad (2)$$

Plugging in the values we get

$$\frac{\alpha R_{io}}{r} - \frac{3R^2 r cos\theta}{2R_{io}^2} = constant \qquad (3)$$

Here, R is the average of $r(\theta)$ taken over the θ variable and has the approximate value of $R = 1.8 * 10^6 (m)$ according to the data regarding Io, and $\alpha = M_{io}/M_j$. Taking the derivative of this equation with respect to θ gives

$$\begin{cases} \frac{dr}{d\theta} = \frac{ersin\theta}{1+ecos\theta} \\ e = \frac{3}{2}\left(\frac{R}{R_{io}}\right)^4 \frac{1}{\alpha} = 1.1*10^{-5} \end{cases} \tag{4}$$

This is the equation of an ellipse with an eccentricity $e = 1.1*10^{-5}$. Thus we see that the shape of the surface of the fluid can be described by an ellipsoid. We can go further to calculate the height of the tides on Io from the fact that we have the core of Io on the focus point of ellipse surface so the difference between $r = \frac{r_0}{1-e}$ and $\frac{r_0}{1+e}$ can give us an estimation on the height of the tides.

$$H_{tide} = \frac{r_0}{1-e} - \frac{r_0}{1+e} = 68.6(m) \tag{5}$$

which is close to other estimations made in the articles cited.

III. Temperature Difference Between a Solid Sphere and a Fluid Moving Past

Heat transfer in an incompressible fluid is given by the equation [4]

$$\frac{\partial T}{\partial t} + \vec{v}.\nabla T = \chi \nabla^2 T + \frac{\nu}{2c_p}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)^2 \tag{6}$$

In this equation indices are summed according to the Einstein's summation rule. Here, T is the temperature, $\vec{v}$ is the velocity, $\vec{x}$ is the space coordinate, and χ and ν are thermometric conductivity and kinematic viscosity of the fluid respectively.

Let us assume that Reynolds number of our fluid is small, and quasi-static equilibrium is reached.

$$\vec{v}.\nabla T \ll \chi \nabla^2 T \tag{7}$$

$$\frac{\partial T}{\partial t} = 0 \tag{8}$$

So that Eq.6 takes the form

$$\chi \nabla^2 T = -\frac{\nu}{2c_p}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)^2 \tag{9}$$

For this case, we first need to find velocity of the fluid as a function of coordinates. To calculate this it is assumed that we are dealing with a incompressible fluid. We write the velocity from equation (20.9) in [4] for spherical symmetry.

$$\vec{v} = -\frac{3}{4}R\frac{\vec{u} + \vec{n}(\vec{u}.\vec{n})}{r} - \frac{1}{4}R^3\frac{\vec{u} - 3\vec{n}(\vec{u}.\vec{n})}{r^3} + \vec{u} \tag{10}$$

Where R is the radius of the sphere moving in the fluid, and u is the sphere's velocity. Plugging this expression for velocity of the fluid to Eq.9, we get a differential equation for T which has a solution in the form

$$T = f(r)cos^2\theta + g(r) \tag{11}$$

where $f(r)$ and $g(r)$ are found from boundary conditions. Calculating these functions we get the temperature difference between the solid sphere and the fluid.

$$\Delta T = \frac{5u^2 P}{8c_p} \tag{12}$$

where u is the mean speed of the fluid, P is the Prandtl number, and c_p is the heat capacity per mass.

IV. The Model

The main concept governing our model is that we take viscosity of fluids surrounding Io as the primary cause of heat production, which is in line with the recent findings on Io's volcanoes' placement. In the previous section we looked at a solid sphere moving in an incompressible fluid which is essential for our model. As we derived earlier the shape of the fluid surrounding Io can be taken as a ellipsoid which has semi-axises that depend on the distance between Io and Jupiter. This distance changes overtime as a result of the eccentricity of the orbit, so that we have a fluid shell moving relative to Io's solid core. We approximate this relative motion to get a averaged constant velocity. We add a solid core of radius $R = 1.8*10^6(m)$ to the fluid ellipsoid model that we have previously calculated. From our previous calculations this ellipsoid's surface projected to the plane of the orbit can be written as

$$r = \frac{p}{1+\epsilon cos\theta} \tag{13}$$

where p is a constant that depends on the radius of Io. The calculations that we did in the first section show that

$$p = \sqrt{\frac{M_{io}}{M_j}}R_{io} \tag{14}$$

$$\dot{r} = \epsilon \alpha^{\frac{1}{2}} \frac{2\pi}{T} Rsin(\frac{2\pi}{T}t) \tag{15}$$

where T is the period, R_{io} the average distance between Jupiter and Io, and ϵ is the eccentricity of Io's orbit. Taking the root mean square of $\dot{r}$, we get

$$u = \sqrt{2}\frac{\pi}{T}\alpha^{\frac{1}{2}}R \tag{16}$$

Let us assume that composition of the fluid covering Io is %20 magma and %80 water. It is known that magma's Prandtl number is around 10^3 and water's Prandtl number is around 6.74. Heat capacity for water $c_p = 4.2(J/g.K)$ and for magma $c_p = 1.0(J/g.K)$. Taking the weighted averages of these values and plugging them in Eq.12 we get

$$\Delta T = 5.1(K) \tag{17}$$

It is known that the heat flux between a fluid and solid is of the form

$$q = h\Delta T \tag{18}$$

Here $h = N\kappa$, where N is the Nusselt number and κ is the conductivity of the fluid. It can be shown that this number depends on Rayleigh number as shown.

$$N = \frac{0.14R_{aL}^{\frac{1}{3}}}{L} \tag{19}$$

Here L is the characteristic length of the system for which we take the thickness of Io's crust that is estimated to be around $50(km)$. R_{aL} is the Rayleigh number that we calculate as follows

$$R_{aL} = \frac{\rho^2 g\beta L^3 \Delta T' c_p}{k\eta} \tag{20}$$

Taking these values' averages for magma and water we get approximately.

Table 1: *For a fluid with %20 magma and %80 water composition.*

k=	2.1328	$\left(\frac{W}{m.K}\right)$
β=	188	(K^{-1})
g =	1.79	$\left(\frac{m}{s^2}\right)$
$\Delta T'$ =	1400	(K)
ρ=	3.5	$\left(\frac{g}{cm^3}\right)$
c_p=	3.35	$\left(\frac{J}{g.K}\right)$
η=	10^{14} to 10^{15}	$(Pa.s)$

where ρ is the density of the fluid, g is the acceleration due to gravity, β is the thermal expansion coefficient, $\Delta T'$ is the temperature difference between fluid's two different boundaries, η is the dynamic viscosity, and k is the thermal conductivity. Calculating for $h = N\kappa$ gives

$$h = 0.628\left(\frac{W}{m^2.K}\right) \tag{21}$$

From which we get the heat flux that is heating Io as

$$q = h\Delta T = 3.20\left(\frac{W}{m^2}\right) \tag{22}$$

which is in the range of $2.4 - 4.1$ that is suggested by other articles cited.

V. Summary

Latest theoretical studies [7] show that Io has a 'magma ocean' around itself. Therefore, in this paper, we took the Jovian moon Io as a combination of a solid core and a viscous fluid constantly oscillating around the spherical solid core. To give a rough estimate to the power output, we calculated the heat flux between the fluid and solid parts by assuming that Reynolds number of the fluid is small. Then, by taking the time-average of the velocity of fluid at the sub-Jovian point, we estimated the heat output at this point to be $3.2W/m^2$, in the range of the result found in the study [2]: $3.8W/m^2$. The fairly accurate result we obtained indicates that our simple yet progressive model was indeed, well-suited to our purpose of finding the related parameters of the extreme tidal heating of Io, the most volcanically active body in the Solar System [8].

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