Irvine University High School

Physics club Team 1

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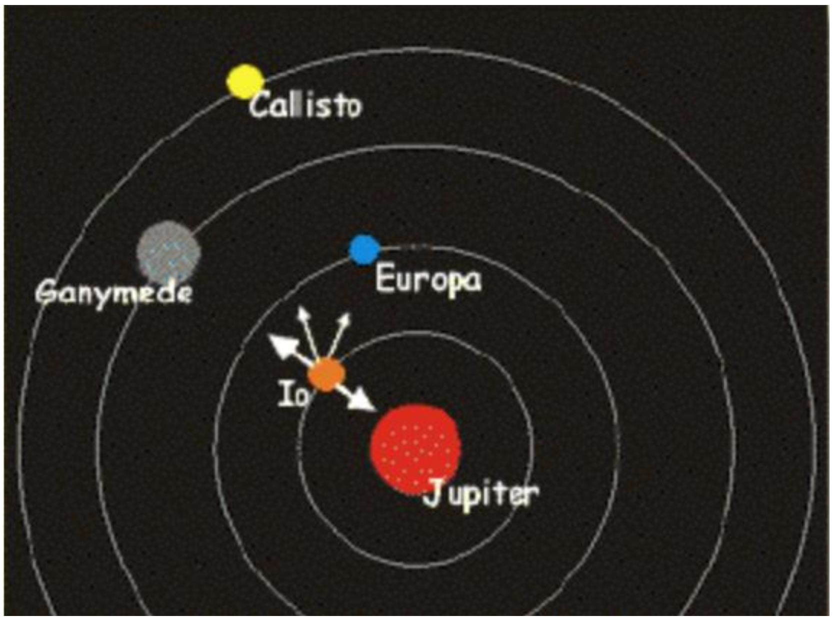
Tidal Heating



a. The deformation of the Io

Tidal heating is the heating of the interior of one planetary body caused by stresses induced from the gravitational pull of another. Io, a tiny moon which orbits very close to the giant planet, is very strongly affected by the pull of Jupiter's gravity. There are other moons nearby which exert a gravitational pull of their own, which also need to be considered. Io's volcanic activity is caused by the powerful force of Jupiter's gravity, coupled with the gravitational pull of Io's neighboring moons--Europa, Callisto, and Ganymede. Jupiter pulls Io inward towards itself, while the gravity of the outer moons pull it in the opposite direction. These opposing forces cause the distance between Io and Jupiter to vary, making Io's orbit slightly elliptical. As a result, Io is subjected to tremendous tidal forces that alternately squeeze and stretch its interior. This causes Io's surface to rise and fall by about 100 meters (300 ft).

Parameter	Io	Europa	Ganymede	Callisto
$m(10^{22}\ \mathrm{kg})$[7]	8.9319	4.7910	14.817	10.762
$R_e(\mathrm{km})$[7]	1826.1	1561.3	2634.5	2408.8
$J_{20}(10^{-5})$	186	43.83	12.73	3.50
$C_{22}(10^{-6})$	559[1]	131.5 [3]	38.18[2]	10.5 [4]
$a(10^5\ \mathrm{km})$[7]	4.216	6.709	10.70	18.83
$q(10^{-5})$	171.3	49.78	19.10	3.694

$$F_t = \frac{GMm}{R^2 - r^2} - \frac{GMm}{R^2} \approx \frac{2GMmr}{R^3} .$$

b. The internal structure and the magnetic field

In the solar system, there are only two kinds of planets, terrestrial planets and jovian planets. Comparing the two sorts of planets, the the terrestrial planets have solid surfaces, while the jovian planets are made of gaseous surfaces.Second, the jovian planets are much larger than the terrestrial planets. Io has a smaller size relative to other moons of Jupiter. Therefore, Io is possibly a terrestrial planet. The terrestrial plants include Mercury, Venus, Earth, Moon and Mars. The internal structure is supposed to be similar to those planets.

Thus, we can apply Lane-Emden equation for this situation, and some appropriate boundary conditions to solve this problem. Since Io is a terrestrial planet, starting from the equations of hydrostatic equilibrium and mass continuity, we can easily get Lane Emden Equation

$$\frac{1}{r^2\rho}\frac{d}{dr}\left(r^2\rho^{b-2}\frac{d\rho}{dr}\right)(r^2\rho^{b-2}\frac{d\rho}{dr}) + \frac{4\pi G}{K_0}\rho_u^2 = 0$$

At here $\rho_u$is the density of the satellite without pressure($p = 0$), which is the average density. $K_0$is the bulk modulus and b is a constant which varies in the core and mantle. Since Io is a terrestrial planet, it should has approximately a similar structure to that of Earth and Moon. Thus we can get the $K_0$and b that works best with the Earth and Moon:

Zone	K_0(Mbar)	b
Core	2.50	3.50
Mantle	1.10	4.50

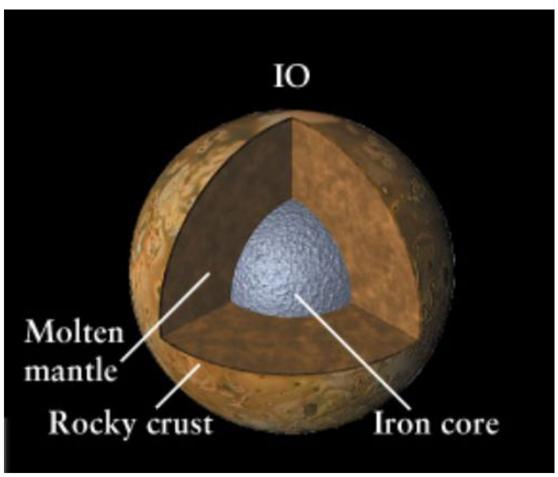
After applying appropriate boundary conditions, we get

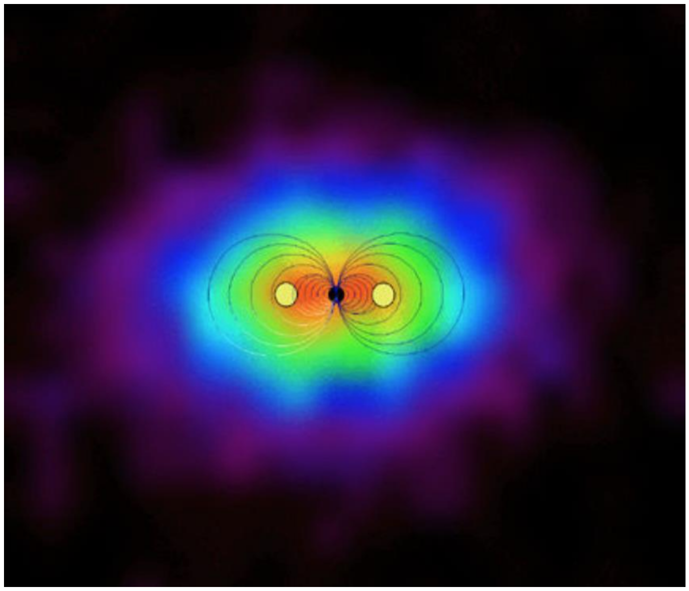
Zone	Parameter	

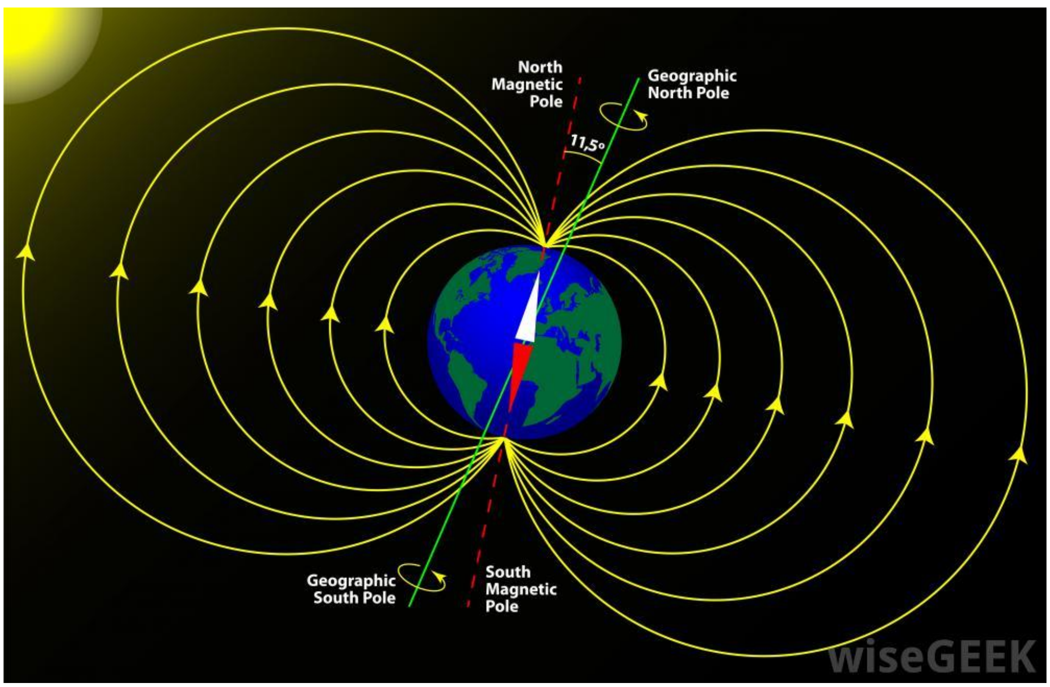
Core	$\rho_C(gcm^{-3})$	4.65
	$R_C(km)$	723.52
Mantle	$\rho_m(gcm^{-3})$	3.3206
	$R_m(km)$	1652.2

At here we simply neglect the thickness of the shell since it is too thin compared to the core and the mantle.

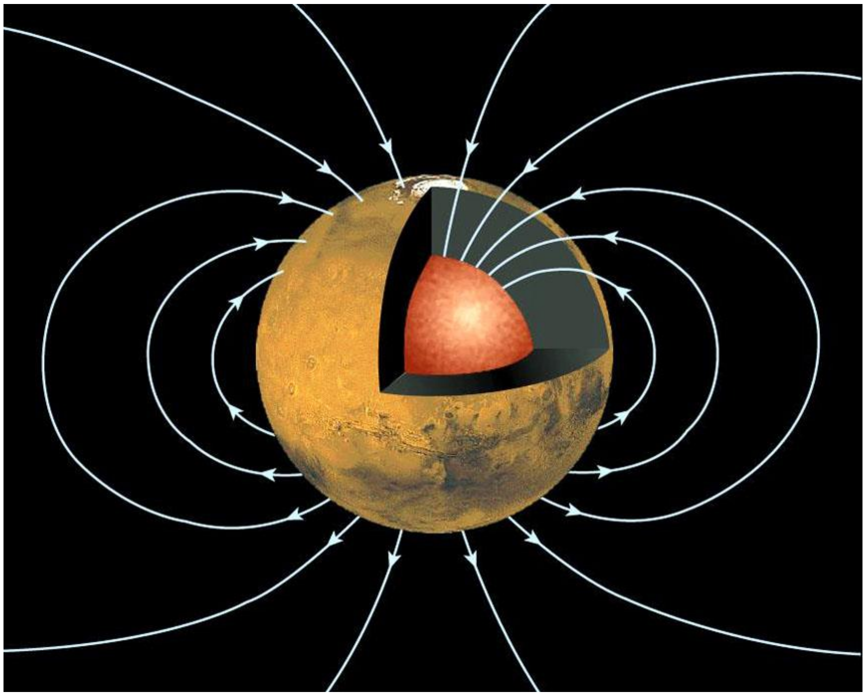
Based on our data, there would be be an Fe or Fe-S rich core in the interior of Io. Outer layers are composed of a olivine-rich crust and a silicate-rich mantle. Because of Io's special interior, the tidal heating is able to process, and forms different distribution of volcano. The eruption of these volcanoes release the sulfur from the interior of Io into the atmosphere, creating an ionosphere which is a good conductor of electric. Because Io orbits Jupiter and intersects with its powerful magnetic-field lines which acts like an electrical generator, some of the sulfur is picked up by Jupiter's magnetic field and accelerate the sulfur to a very high speed. Sulfur as the form of sulfur dioxide is energized by the charged particles from the electrical fields. It creates a radiation belt around Jupiter in the magnetic field.






North
Magnetic
Pole
Geographic
North Pole
11,5°
Geographic
South Pole
South
Magnetic
Pole
wiseGEEK

Io
Earth



Mars

c. Io’s surface change under tides
Plume deposit changes shape and distributions. Additional dark material forms. New lava flow appears. Aurorae created by the interaction between Io's atmosphere and volcanic plumes with the Io flux tube and the plasma torus.
The highest tidal force a body exerts on its moon is the difference of the maximum and minimum forces.
The net gravitational force and the Shear Modulus are considered in the model.
For Jupiter and Io, the highest force is on the near side of the Jupiter when Io is at perigee (average (it's complicated) R_0=420000 km). The lowest force is on the far side of Jupiter when Io is at apogee (average R_1=423400 km). In the equation, M_j is the mass of Jupiter, which is $1.898 \times 10^{27} kg$. m_i is the mass of Io, which is $8.931938 \times 10^{22} kg$. G is the gravitational constant. r_i is the radius of Io, 1821600 m.

$$F_{min} = G\frac{Mj \quad mi}{\left(R_1 + r_i\right)^2}$$

$$= 6.67408 \times 10^{-11} m^3 kg^{-1} s^{-2} \times \frac{1.898\times10^{27} kg \times 8.931938\times10^{22} kg}{(423400000m + 1821600m)^2}$$

$$= 6.3061 \times 10^{22} N$$

$$F_{max} = G\frac{M_j m_i}{(R_0 - r_i)^2} = 6.67408 \times 10^{-11} m^3 kg^{-1} s^{-2} \times \frac{1.898 \times 10^{27} kg \times 8.931938 \times 10^{22} kg}{(420000000m - 1821600m)^2}$$

$$= 6.4197 \times 10^{22} N$$

$$\Delta F = 1.136 \times 10^{21} N$$

Therefore, the tidal force is $1.136 \times 10^{21} N$. The surface of Io can be regarded as a horizontal plate, when we observe the tides on the ground of Io.

$$F_{total} = F_{tidal} \times 2r_i = 1.136 \times 10^{21} \times 2 \times 1821600m$$
$$= 4.1387 \times 10^{27} N$$

The work done by the movement of rock layers is $W = F \times d$. d is the distance that the rock moves. According to our research, $P = 6 \times 10^{17} W$, $t = 146880s$

$$\underline{P} = \frac{\Delta F \times d}{0.5 \times t_i} = \frac{1.136 \times 10^{21} N \times d}{0.5 \times 146880s} = 6 \times 10^{17} W$$
$$d = 38.789m$$

d. Volcanoes

This perpetual friction generates enormous amounts of heat and pressure within Io, causing molten material and gases to rise through fractures in the crust and to erupt onto the surface.

e. Energy source

Since Io is too small to have left over accretional heat, and radioactive decay could not generate the tremendous energy required to power all of the volcanic activity that exists on the moon, it is the tidal heating resulted from the gravitational forces of Jupiter and others of Jovian moons that is responsible for powering the planet. At the end due to the work done by the friction inside the satellites, the satellites will get closer to Jupiter and eventually collide into Jupiter.

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