Tidal heating Model

1. Some basic descriptions about the energy exchange model of the system

1.1 Two ways of tidal heat generation and their energy exchanges

Tidal heating on Io is a pretty special case among the whole solar system. It has got some similarities to Earth-Moon system, but the special orbital resonance makes it a totally different picture when presented to observers like humans ourselves. What causes this special phenomenon is the constant tidal heating. There are basically two ways to generate tidal heat by the strong and strange tidal force:

i. The difference between the period of the moon's self-rotation and the period of the moon's orbit around the planet, which could cause friction (the mechanism which generate heat) when difference in these two periods exists.
ii. The eccentricity, which results in the constant stretch and squeeze of the moon, generating a large amount of heat.

Although these two mechanisms can both generate tidal heat and are both caused by tidal forces, the energy transfer is somewhat different.

In the first scenario (tidal locking), the heat is transferred mainly from the rotational energy of the body being affected by friction caused by the tidal force. In other words, it is the rotational energy that has been transferred to heat and orbital energy since the conservation of angular momentum and the conservation of energy should be applied here. Here's how we obtain this piece of information:

Take the moon's orbital radius as $\mathbf{R_{orbit}}$ ($\mathbf{R}$ to be simpler), the moon's radius as $\mathbf{r_{moon}}$ (r to be simpler), the moon's mass as **m**, and the moon's angular velocity around the orbit as **w.**
As we have been aware that the moon is tidal locked to the planet, the angular velocity of the moon moving around the orbit is the same as its own angular velocity of rotation. Therefore, the angular momentum of the moon's rotation and its orbit can be expressed as:

$$\mathbf{L=mR_{orbit}^2w+2mwr_{moon}^2/5.}$$

Since $\mathbf{w}=\sqrt{\frac{\mathbf{GM}}{\mathbf{R^3}}}$, where M is the mass of the center body, we can rewrite the above expression as

$$\mathbf{L} = \mathbf{m}\sqrt{\mathbf{GMR}} + \frac{\mathbf{2m}\sqrt{\frac{\mathbf{GMr^4}}{\mathbf{R^3}}}}{\mathbf{5}}.$$

Take the first derivative of L with respect to R we can acquire:

$$\frac{\mathbf{dL}}{\mathbf{dR}} = \frac{1}{2}\boldsymbol{m}\sqrt{\boldsymbol{GM}}\boldsymbol{R}^{-\frac{1}{2}} - \frac{3}{5}\boldsymbol{r}^2\boldsymbol{m}\sqrt{\boldsymbol{GM}}\boldsymbol{R}^{-\frac{5}{2}}.$$

When the derivative is larger than 0, the total angular momentum of moon will increase as it moves further away from its planet.

Calculating the result, we will get that when $\mathrm{R} > \sqrt{\frac{6}{5}}r$, the situation described above will become a true fact. Since the orbit radius is far larger the radius of a moon, we can confirm that during the first scenario (tidal locking) that the rotational energy is what is transformed into heat and orbital energy and the moon and the planet will move further away from each other.

In the second scenario (tidal circularization), the heat is transformed mainly from the orbital energy of the body whose orbital eccentricity is nonzero. This still follows the law of conservation of angular momentum. **Imagine** that one day the Jupiter is tidal locked to its moon (although this cannot happen since sun will engulf Jupiter before this could be realized) and the moon's orbital eccentricity is nonzero, the rotational angular momentum and the orbital angular momentum of the moon decrease as it moves to a lower orbit. This is exactly the **reverse** process as the first scenario. Tidal circularization consumes orbital energy and accelerate the spin rate of both the planet and the moon (on the premise that Jupiter is tidal locked, we only look at one of the factors before combining them). Since the Jupiter is tidal locked, the only energy source must be the moon's orbital energy, which transfers part to heat and part to Jupiter's rotational energy (the angular momentum of Jupiter's rotation also increases as the total angular momentum of the moon decreases). Therefore, this scenario is also reasonable when applying the known laws of physics.

1.2 How the two tidal heat generation mechanisms work for Io system

So combine these two scenarios we obtain the special case of one of the Jovian moon—Io. On one hand, the eccentricity of Io's orbit would like to decrease to 0, generating heat and lowering Io's orbit; on the other hand, the difference between the period of Io's orbit and that of Jupiter heats up Jupiter and provides orbital energy to Io. Therefore, it seems that Io's orbit should raise (as Jupiter loses its angular momentum, its moon gains it, thus $\mathbf{R_{orbit}}$ increases) and gradually becomes a circle. However, the thing that greatly differentiate Io from other scenarios happening in the solar system is that it just keeps its slightly oval orbit.

Therefore, we need to take into account the other moons——Europa and Ganymede. Currently it is said that Io, Europa, and Ganymede are in 1:2:4 orbital resonance, or Laplace resonance. This is a relatively stable system. Here's how the energy flow through the system:

The three moons are all in the slightly eccentric orbits around Jupiter, thus following the second mechanism of heat generation as described above (tidal circularization). But the interaction prevents them to become more circular. Therefore, they will consume their orbital energy at the same pace which help maintain 1:2:4 resonance. Same as the scenario of single Io-Jupiter system, this scenario also indicates that Jupiter will convert its own rotational energy to heat and orbital energy of all of the three moons, thus supporting Io, Europa, and Ganymede to maintain and even raise their orbits. In short, by combining all these details, the energy source which eventually heats up Io, Europa, and Ganymede is Jupiter's rotational energy. Although transforming indirectly, the rotational energy of Jupiter indeed heats up the moons and repel them further away from Jupiter (Further information about the future of Jovian moons will be discussed more detailly in the following chapters).

We will be using these energy details as the bases of our model to analyze Io system.

2. Force model of the Io system and its implications on Io

2.1 Tidal force on Io and its deformation

Whatever celestial bodies experience tidal forces, and they can be significantly larger when the center body's mass is huge and the distance between those two bodies is quite small. This is especially true for Io. Tidal force is known to be calculated as the derivative of the function of gravitational force with respect to the distance between the center object and the object with moves around the center object, as calculated in the following expression:

$$\frac{\mathbf{dF}}{\mathbf{dR}} = -\frac{2GMm}{R^3}.$$

Where F is the gravitational force between the two celestial bodies, R is the distance between these two bodies, G is the universal gravitational constant, M is the center body's mass, and m is the mass of the body moving around the center. Therefore, we can see that Io can be significantly distorted due to the big M and small R.

Since Io's orbit is not perfectly circular, it just informs us that there is one near-side spot (periapsis) and one far-side spot (apoapsis). So, it can be reasonably inferred that when Io moves around Jupiter, it will become more oval when nearer and more circular when farther. When Io completes one circle, it will be squeezed for one time, and stretched for one time, which can be illustrated in the following

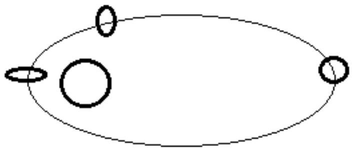

figure:

Figure 1: Io deforms in the gravitational field of Jupiter (exaggerated)

2.2 An estimation of the heat generation as a result of tidal forces

As Io rotates around Jupiter, due to its orbital resonance with Europa and Ganymede, Io's orbital eccentricity cannot decrease as it will probably behave if without the interactions with the other two moons. Therefore, the tidal force will constantly do work on Io, making it generate heat constantly.

Since Io is in orbital resonance, which means that during one period of time, the eccentricity of the moon's orbit will not change, the energy transformed can only be the moon's rotational kinetic energy, heat, and its own orbital energy (Although Jupiter provides orbital energy to Io which exceeds its own rate of consumption). Therefore, we can possibly derive an expression for the amount of heat generated within half of the period of time when Io moves from the apoapsis to the periapsis.

The two extreme tidal forces that Io experiences can be calculated in the following way:

The tidal force at the apoapsis= $-4GMmr/R_{apoapsis}^3$,

The tidal force at the periapsis= $-4GMmr/R_{periapsis}^3$,

where $R_{apoapsis}$ is the distance between the moon and Jupiter when the moon is at apoapsis. The same also applies to $R_{periapsis}$. Since constant force cannot change the shape of Io anymore, thus it cannot induce heat within Io. The force that really does work to Io is the difference of these two tidal forces:

$$\Delta F = 4\left(\frac{\mathbf{GMmr}}{R_{apoapsis}^3} - \frac{GMmr}{R_{periapsis}^3}\right).$$

This difference of forces will do work for an amount of distance which can be obtained by observing the tidal height of Io. The tidal height is primarily caused by the different tidal forces of Jupiter, but it is also caused by the other two moons—Europa and Ganymede. It is two effects that contribute to the large amount of tidal height observed on Io.

Assume $\mathbf{\Delta h}$ as the tidal height. Since the components of $\mathbf{\Delta F}$ does not move for equal distances. The component on the outer edge move for the full amount, the inner most move a distance close to 0. Therefore, the total work done by Jupiter is approximately the product of the forces' difference and half of the tidal height, which is:

$$\mathbf{W}=\frac{\mathbf{\Delta F \Delta h}}{\mathbf{2}}.$$

After we have approximated the work done by the tidal force, we will have to calculate the change in the potential energy of Io itself. Since Io's shape changes all the time, as the tidal force pulls Io to the shape of an elliptical, the work done will transform part to the rocks' potential energy on Io and part to heat due to friction between the rocks. The change of potential energy of these moved rocks can be calculated as follows:

Firstly, here is a figure illustrating the change of Io's shape when it moves from its apoapsis to periapsis:

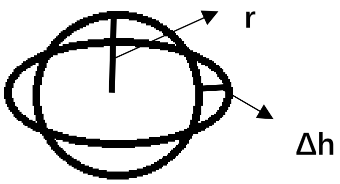

Figure 2: Io deforms in Jupiter's Gravitational field, which is stretched on one direction and squeezed on the other direction while keeping its volume

Assume the radius of Io at the apoapsis is **r**. As indicated above, we can potentially utilize the tidal height Δh to calculate the change in potential energy of the rocks that are moved by the tidal force. Since the volume will not change and the length of the semi-axis perpendicular to the direction of tidal force and the horizontal plane will also not change, the near-circle and the oval (exaggerated) should have the same area. Therefore,

$$\mathbf{\pi AB = \pi r^2},$$

where A and B are respectively the oval's length of semi-major axis and semi-minor axis.

Since **A=r+Δh**, we should have

$$\mathbf{B} = \frac{\mathbf{r^2}}{r+\mathbf{\Delta h}}.$$

The common area of these two circles can be approximated as an oval. Assume the length of the semi-major axis is **a** and the length of semi-minor axis is **b** with

a=r and b=r-B.

Therefore, the amount of potential energy change for a single plane is the difference of the potential energy change of the rocks raised and that of the rocks dropped, which is

$$\Delta E_{potential-plane} = a_g m_{rocks} \Delta H = \frac{GM}{R^2} m_{rocks}[\Delta h - (\mathbf{r} - \mathbf{B})] =$$

$$\frac{GMm_{plane}\Delta h^2(r^2-Br)}{r^5+\Delta h r^4},$$

where M is the moon’s mass.

So, if we cut Io into countless planes like the one we are dealing with above, we can make a summation of the total potential energy changes:

$$\Delta E_{potential-total}=\frac{\sum_{i=1}^{n} GMm_i\,\Delta h_i^2\left(r^2-Br\right)}{r^2\left(r_i^3+\Delta h_i r_i^2\right)}\approx\frac{GM^2}{r^2}\frac{\Delta h^2\left(r^2-Br\right)}{r^2(r+\Delta h)}$$

Eventually, the heat generated **ΔH** is the difference of **W** and $\Delta E_{potential-total}$, and the rate of heat generation **P**$_{\mathbf{heat}}$ is quotient of the heat generated and the time spent:

$$p_{heat}=\frac{\Delta H}{\Delta t}=\frac{2\left(W-\Delta E_{potential-total}\right)}{T}$$

$$=\frac{4GMmr\left[\Delta h\left(\frac{1}{R_{periapsis}^3}-\frac{1}{R_{apoapsis}^3}\right)\right]-2\sum_{i=1}^{n} GMm_i\,\Delta h_i^2(r^2-Br)/r^2(r_i^3+\Delta h_i r_i^2)}{T}$$

$$\approx\frac{4GMmr\left[\Delta h\left(\frac{1}{R_{periapsis}^3}-\frac{1}{R_{apoapsis}^3}\right)\right]-2\frac{GM^2}{r^2}\frac{\Delta h^2(r^2-Br)}{r^2(r+\Delta h)}}{T}$$

This is the estimated rate of heat generation on Io. We can further complete this equation by calculating **Δh** (which will need $K_{2,0}$ love number, Q_0 love number, and Io’s orbit’s eccentricity). But since we have observed the tidal height changes happening on Io, this approximate way of heat calculation may be proposed.

2.3 How heat dissipates on Io

Since the heat is generated, this huge amount of heat must go somewhere. Primarily, the heat first heats up the inner layer of rocks, which possibly has melted a significant amount of mass of rocks and raised those rocks’ temperature. Next, the molten rocks with such high temperatures will transfer heat through geologic activities——especially the volcanism. Heat will be dissipated among Io’s surface.

Eventually, through convection with the vacuum space, heat is lost to Io’s surrounding environment. This process can be illustrated in the following figure:

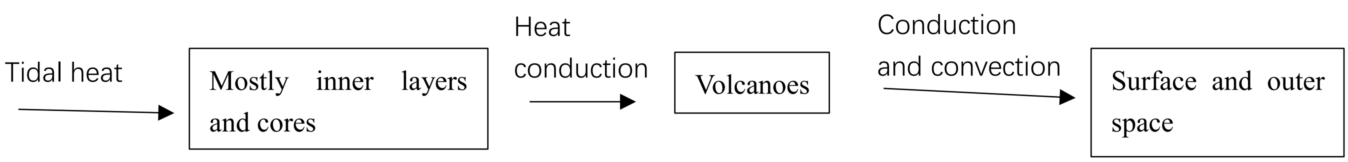

Figure 3: how heat flows through Io

Therefore, through the process of volcanism, Io's surface receives a significant amount of heat. Through the electro-magnetic spectra obtained from Io's surface (the extreme intensity mainly appears around 16~22[1]µm), we can get a hint at the surface temperature. According to Wien Displacement Law, **λT=b**, we can calculate the value of T which is approximately 131~181K. But near the site of volcanoes, the temperature will be far higher than that.

3. Model of Io's internal structure

3.1 General model of Io's interior

Io experiences pretty large tidal forces, and this is similar to some of the other celestial bodies in the solar system. Mercury is similar to Io's situation (but large differences between these two bodies exist as well), experiencing large tidal forces from the sun. But it does behave very active volcanic activities. It is also reported that Mercury and Io has got similar materials compositions——mainly silicate and iron. Other bodies like Mimas, one of Saturn's moons, also experience huge tidal force from its planet. However, its composition is far different from Io, mainly consisting of ice, which cannot generate so much heat as Io does. Therefore, we can tell that the reason why Io is so different from other planetary bodies is its appropriate compositions and its driving force (the tidal heating).

Firstly, we will have to deal with Io's mass, volume, and its main compositions within the crust. To simplify the model, we will only assume the main composition of Io is silicate and iron. Given that the iron's density is $7.9g/cm^3$ (for liquid iron, the density is about $7g/cm^3$, but we will first assume that iron inside Io is mostly solid) and most silicate's average density is $3.3g/cm^3$, we can calculate the approximate volume of silicate $V_{silicate}$ and that of iron V_{iron}, which is

$$V_{silicate}:V_{iron} \approx 7:1,$$

With $V_{silicate} \approx 2.2*10^{10}\ km^3$ and $V_{iron} \approx 0.3*10^{10} km^3$.

Since iron mainly deposits around the core, silicate mainly deposits in the mantle and the crust, the increase of density of iron will be slightly higher than that of silicate, making the ratio of the volume of iron to that of silicate higher. Generally, it can be estimated that the core radius takes up approximately 30% of the total radius of Io, with main composition consisting of Iron and other heavy metals. The other 70% is mainly silicate with some metals like iron, which form the mantle and the crust.

[1] William M. Sinton, "The thermal emission spectrum of Io and a determination of the heat flux from its hot spots "

3.2 Io's inducted magnetic field and its implications

As we have noticed in section 3.1, it is the right condition (tidal heating) and special composition (iron and silicate, which give Io the largest density of all moons in the solar system) that contribute to Io's intense volcanic activity. This intense activity indicates that Io's interior has a significant amount of molten rocks, which is pretty different from mercury, since mercury contains even more iron than Io. The presence of large amount of silicate contribute to Io's active interior. However, this is not all Io's amazing aspects. Io's iron and its right amount also have a big impact on the magnetic field of Jupiter.

How Io moves through Jupiter's magnetic field can be illustrated in the following figure:

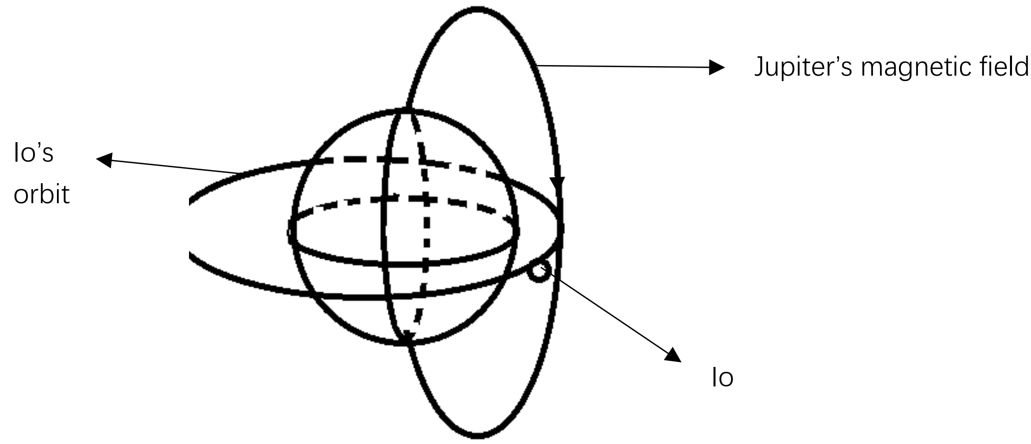

Figure 4: Io moves through Jupiter's magnetic field and generate electricity

Statistics taken from the probes sent has indicated that Io generate electricity at an amazing rate of 1 MW, creating approximately 400000 volts of electrical potential[2]. Since Io does not generate magnetic field itself. This is the inducted magnetic field and it greatly indicates the existence of molten rocks with iron within the layer. The existence of molten iron and Jupiter's magnetic field together creates enormous ring currents inside Io.

4. The future of Jovian moons

As we have once mentioned in section 1 about the energy flow throughout the Io-Europa-Ganymede system, whether or not these moons will eventually move farther away from Jupiter or closer towards it really depends on how we observe the heat generated by the two ways we discussed (which are the existence of eccentricity and the difference in the rotational and orbital period of the moons). These two ways of generating heat can cause the opposite outcomes.

2 Schneider, N. M.; Bagenal, F. (2007). "Io's neutral clouds, plasma torus, and magnetospheric interactions"

Currently, through different observations, we have seen a lot of small Jovian moons move further away from Jupiter, which greatly indicate that Jupiter is still constantly losing its angular momentum which transforms into the angular momentum of the other moons. Therefore, it can still be reasonably inferred that Io, Europa, and Ganymede will move further away from Jupiter as the other moons, since they experience the same mechanisms.

However, this is not the whole story. As we have seen, Io, Europa, and Ganymede have formed a stable Laplace Resonance for a very long time. These moons also constantly generate heat. Currently they move further away from Jupiter because the orbital energy given by Jupiter exceeds the consumption of their own orbital energy, thus it propels them to a higher orbit. But when their orbits get higher and higher and their orbital period gradually matches the period of Jupiter's rotation, the heat generated by Jupiter (as well as the orbital energy provided) will gradually decrease. At a certain point, the orbital energy will no longer be able to support the other three moons. But these moons will still be able to constantly generate heat. Therefore, their orbits become to get lower, consuming their own orbital energy to generate heat, and accelerate the rotation of Jupiter once again.

So, these two situations can both happen possibly, depending on the amount of rotational kinetic energy of Jupiter left. But through estimations, Jupiter can still support these Jovian moons for a time pretty longer than the life time left to our sun. Therefore, the Jovian moons will continue to move away from Jupiter until Sun engulfs all these amazing celestial bodies that once have existed in the universe.